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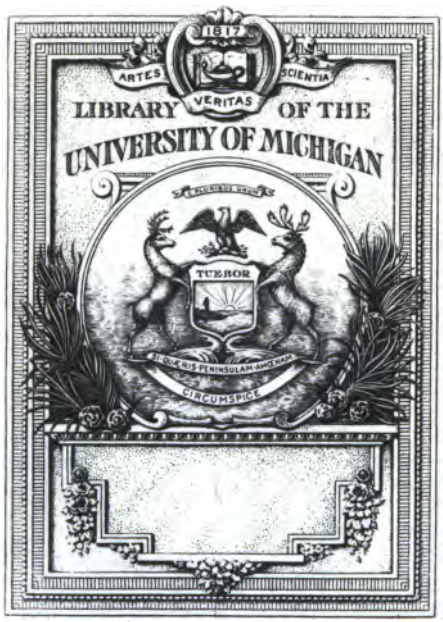
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THE

LEEDS CORRESPONDENT,

A

LITERARY, MATHEMATICAL, AND PHILOSOPHICAL,

Miscellany;

CONSISTING OF

ORIGINAL ESSAYS AND POETRY;

ANECDOTES;

TRANSLATIONS OF LATIN AND FRENCH EXTRACTS;

MATHEMATICAL QUESTIONS

AND ANSWERS, &c. &c.

The Mathematics in the three early numbers edited by

THE LATE

MR. JOHN GAWTHORP,

AND IN THE FOURTH NUMBER

BY

JOHN WHITLEY.

Tarda fluunt ingrataque tempora, quæ spem
Consiliumque morantur agendi gnaviter id, quod
Æque pauperibus prodest, locupletibus æque;
Æque neglectum pueris senibusque nocebit. HORAT.

VOL. II.

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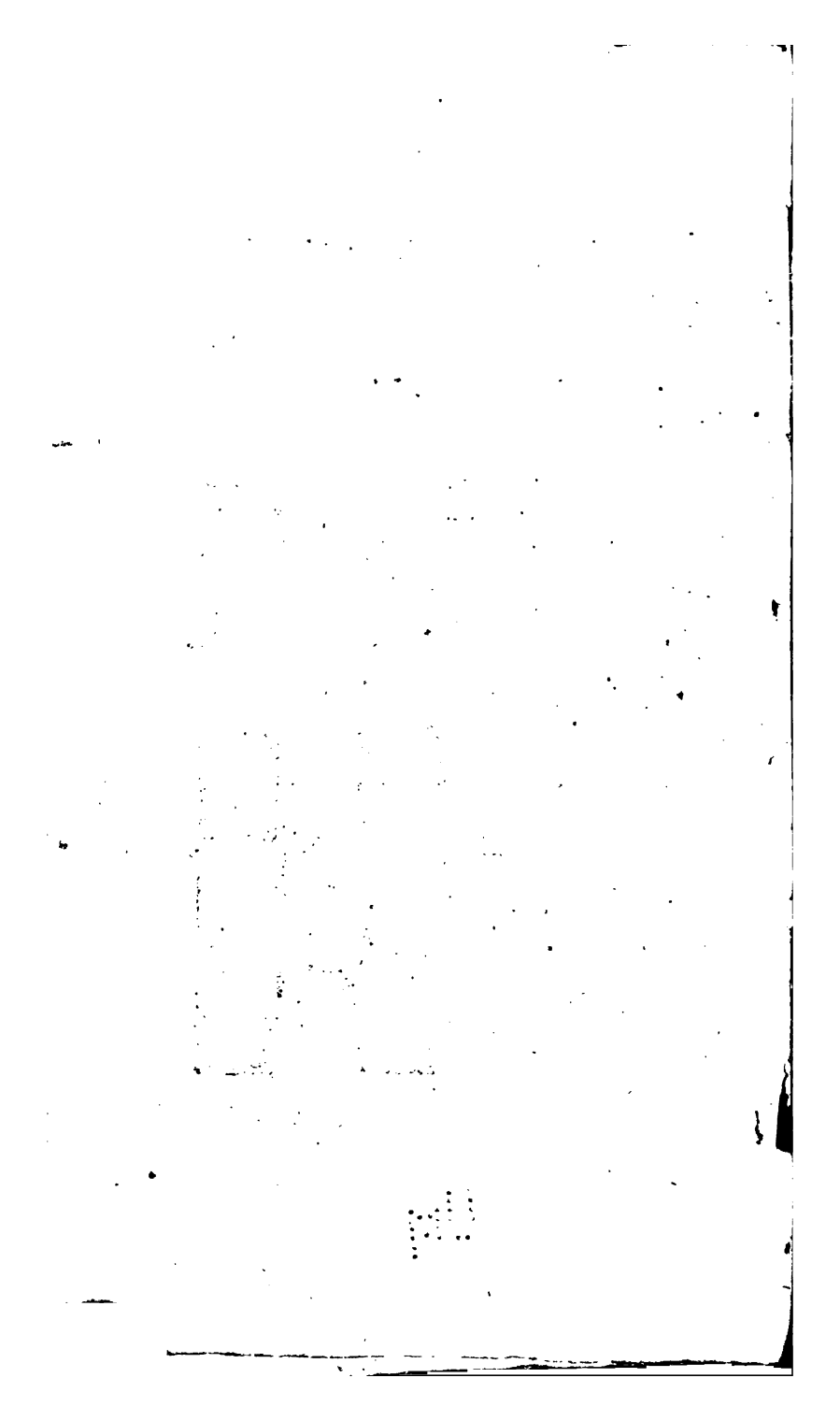

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PREFACE.

NUMEROUS and anxious enquiries have long been made about the continuance of the LEEDS CORRESPONDENT, and the reason why the publication of it has been some time suspended.

AN unusual fatality has attended the execution of this second volume.—When the first volume was nearly completed, the world was deprived of that able mathematician and honest man, Mr. John Ryley, who appeared among the earliest projectors of this work, and was the first editor. In the second and fourth numbers, an attempt has been made to do justice to his memory.—Immediately previous to the appearance of the third number, Mr. John Gawthorp, another mathematician of great promise, was cut off in the prime of life. He had succeeded Mr. Ryley in the revision and management of our Mathematical department.—And while the materials of the fourth number were in a course of preparation, I was involved in misfortune and temporary ruin.

THE particulars of this affair are known to many of the inhabitants of Leeds; and have elicited expressions of condolence and sorrow from multitudes of disinterested persons. It is generally understood that my late bankruptcy was not the effect of idleness, dissipation, or want of encouragement. Neither was it occasioned by an inability to meet all my lawful demands The real cause was a circumstance quite unconnected with business.——The consequent variableness in some men's affections, the maliciousness of others, the misrepresentations and garbled statements of a few, and the undissembled satisfaction of the pragmatical and envious,—are nothing more than the usual concomitants of all similar calamities. To recount them as permanent grievances, or to relate my personal and domestic sufferings, would on my part be an affectation of sentimental writing, which is a sort of distaff that ill becomes the hand of a man. Should the conduct of the small number of those who have been my traducers, continue as violent as it

has been, and their statements be as distant from the truth, it will then become a point of duty with me to enter into such public details as will effectually refute all calumny. But to bear unmerited reproaches without complaining, requires no great exertion of courage: It is a course which I have viewed, as, in many cases, much more manly than the most impassioned yet correct display of wrongs. My practice has always been that of making many and great allowances, when people are known to speak from partial views or interested motives.

When all these things are taken into consideration, when I find myself once more engaged in those literary pursuits which were at all times my delight, when I am favoured with the possession of the very types and printing materials which I formerly used in publishing the Leeds Correspondent,—the respected readers of this Miscellany may form some slight conception of my varied feelings on being enabled to present to them the fourth and concluding number of this volume.—It would be criminal in me to omit this opportunity of publicly ascribing to the good Providence of God, all my support in adversity, every act of kindness on the part of my friends, my speedy re-establishment in business, the encouragement which I have since received, and my very gratifying prospects of success. On this topic I may joyfully add, "Thou holdest up my goings, and maintainest my lot."

And now, if, in treating of myself at this time, I have written more than I ever before wrote or spoke on the same subject, I may, for many obvious reasons, obtain forgiveness. Those who know me best, will, of all readers, be the most surprised at my having written so much on my own affairs: They will, however, believe that a deep consciousness of its necessity and importance is the only motive that could have drawn from me an effort of this kind, which is made with the greatest reluctance. To some worthy and well-affected persons I could easily produce matter of further exculpation for having published these paragraphs, with such examples in vindication

of this practice on proper occasions, as, I am sure, would at once procure for me their indulgence and approval.

The title-page of this volume conveys the information, that Mr. John Whitley, 'of Masbro' near Rotherham, has undertaken the province of Mathematical editor. Till the period of Mr. Gawthorp's indisposition, I had no personal acquaintance with Mr. Whitley. But I had been taught for many years to regard him as a most accomplished mathematician; and I account the accession of such an able auxiliary a grand acquisition to this work, and one that greatly enhances its value. In this opinion of him and his attainments, I am supported by the convincing testimony of the late Mr. Ryley, who was not at all backward in giving utterance to a far stronger declaration than this in Mr. Whitley's favour.

Upwards of two years ago, many of my friends suggested the propriety of enlarging the literary department of the Leeds Correspondent: They thought that the most appropriate change would be that of publishing the work quarterly, and devoting, as usual, one third portion of the January and July numbers to Mathematics, and the two additional numbers for April and October to general literary subjects. Those who had an opportunity of noticing my observation on this proposition in the third number, will recollect that it was my expressed intention to adopt such an arrangement: But my misfortunes at that time prevented me from carrying this plan into effect.—After mature deliberation, I have now resolved to gratify several learned men in suffering the Correspondent to retain its present rank of a half-yearly Mathematical Miscellany; and (in lieu of any alteration,) to commence in January next, 1819, a new monthly work, to be entitled "The Leeds Literary Observer;"—for some account of which, see the Prospectus which accompanies this number.

Communications to both these works, from all our Correspondents, and from as many new friends as we can gain, will be gratefully acknowledged. JAMES NICHOLS.

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HORAT.

No. I. JANUARY, 1816. VOL. II.

Themes and translations are respectfully requested of those, among our junior correspondents, who have not attained to sixteen years of age.

JUVENILE DEPARTMENT.

THEME.—*On true Glory.*

ON THE CRIMINALITY AND BANEFUL CONSEQUENCES OF LYING.

LYING is a vice, which either considered in its own nature, or in the effects it produces, is detestable, and destructive in the extreme. It has been represented by men of all ages, eminent for integrity, wisdom, and virtue, in all its diversified parts, and various tendencies, as an evil which should carefully be shunned, and studiously hated, by all who submit to the exercise of reason. For, of all others, it is a vice which is productive of the most serious and fatal consequences; acting on the mind, in a manner similar to a poison on the body, and expelling from it the sacred principles of truth and reason, which are implanted in the breast of

every man, and by which he is enabled to discern what is truth and what is error. Insolence, sophistry, vanity, obstinacy, and a train of diabolical and destructive passions, are its companions, which universally tend to destroy the credit, happiness, and peace of those who yield to it in place of "heav'n-born Truth."

But not alone to the individuals who basely and unfortunately practise this vice, are its effects mischievous: its malignant influences attack and destroy the bands of social confidence and affection; interrupt the security of life; weaken the credit of intelligence; and, in short, are productive of the most direful consequences imaginable to society in general. It was well observed by Epænetus that "liars are the authors of every species of wickedness and injury in the world." Parasites, calumniators, unfaithful counsellors, and wicked instructors of youth, are accounted by him the most notable proficients in this infernal science. Forgery, bribery, truce-breaking, flattery, party exaggerations, malevolent aspersions, false accusations, and a number of other hateful vices, which are continually harrassing the world, may be traced to their source and foundation in the abominable and heinous vice of lying.

A principle, then, which is productive of such lamentable consequences; exterminating from the mind all that is good, and diffusing all that is evil, must necessarily be highly criminal and reprehensible, by the laws of Him who is truth itself, and "in whom there is no variableness, neither shadow of turning." He has imparted to man the faculty of speech, not for the purpose of deceiving and by smooth words destroying his brother; but that "every man might speak truth with his neighbour." So diabolical is lying, that the devil is represented as "a liar and the father of lying." This vice appears to be one of the habitudes of his mind, a constituent part of his nature, since "when he speaketh a lie, he speaketh of his own." Man is said, by the highest authority in the world, to be earthly, sensual,

and devilish," and therefore the most easily wrought upon by that which is congenial to him in its qualities. But "God is not a man that he should lie, neither the son of man that he should repent." So contrary is this to the Divine Nature, that the Deity is called "God who cannot lie." If his truth was not one of his essential perfections, all the rest would be incomprehensible and fallacious. But his eternal veracity is the great bulwarks of his dominion; and not one of those who have confided in it, has been confounded. Lying is thus proved to be egregiously sinful, because it is an imitation of the works of infernal agency, and directly opposed to Him who is "the way, the truth, and the life," and whose example we are commanded to follow, and to copy his virtues.

It is held in abhorrence, and spoken against, even by those who are themselves adepts in it. "I really know (says the celebrated Lord Chesterfield,) nothing more criminal, more mean, and more ridiculous than lying. It is the production either of malice, cowardice, or vanity, and generally misses of its aim in every one of these views; for lies are always detected, sooner or later." The character of a liar, will also go to prove the atrocious criminality of the principle. "It is (in the words of an eminent writer), so hateful, and contemptible, that even of those who have lost their virtue it might be expected, that from the violation of truth they should be restrained by their pride. Almost every other vice that disgraces human nature, may be kept in countenance by applause and association; but the liar, and only the liar, is invariably and universally despised, abandoned, and disowned; he has no domestic consolation which he can oppose to the censures of mankind; he can retire to no fraternity where his crimes may stand in the place of virtues; but is given up to the hisses of the multitude, without friend and without apologist; for it is the peculiar condition of falsehood, to be equally detested by the good and bad."

It is the indispensable duty, then, of every rational being, to detest and avoid the embraces of a principle, which degrades human nature, and plunges, into the depth of misery and woe, all those who are infected with its baneful contagion, or who act under its diabolical influence. Let it always be remembered, that nothing but strict truth can carry a man through the world, with his conscience and honour unwounded. T. H. MADGE.

Mr. Treeby's Academy, Plymouth.

BIENFAISANCE.

Si j'avois été tant soit peu riche, j'aurois voulu me donner mille jouissances nouvelles; Paris seroit devenu pour moi une autre Memphis. Son peuple immense nous est inconnu. J'aurois eu une petite chambre dans un de ses faubourgs, sur les carrières; une autre, à l'extrémité opposée, sur les bords de la Seine, dans une maison ombragée de saules et de peupliers; une autre, dans une de ses rues les plus fréquentées; une quatriéme, chez un jardinier, dans une maison entourée d'*abricotiers*, de *figuiers*, de *choux* et de *laitues;* une cinquième, dans les avenues de la ville, chez un vigneron, etc.

Il est, sans doute, facile de trouver, partout des logemens de cette espèce à bon compte; mais il n'est pas si aisé d'y trouver des hôtes et des voisins qui soient des honnêtes gens. Il y a beaucoup de corruption dans le petit peuple; mais il y a plusieurs moyens d'y reconnoître les gens de bien; c'est par eux que je commence les recherches de mes plaisirs. Nouveau Diogène, je m'en vais à la quête des hommes. Comme je ne cherche que des malheureux, je n'ai pas besoin de lanterne. Je me lève au petit point du jour, et je vais à une première messe, dans une église encore à demi obscure; j'y trouve de pauvres ouvriers, qui viennent prier Dieu de bénir leur journée. La piété, sans respect humain, est une preuve assurée de probité:

l'amour du travail en est une autre. J'apercois, par un temps de pluie et de froidure, une famille entière couchée sur la terre, en sarclant les herbes d'un jardin: voilà encore des gens de bien. La nuit même ne peut celer la vertu. Vers la minuit, la lueur d'une lampe m'annonce, par les lucarnes d'un grenier, quelque pauvre veuve qui prolonge ses veilles, afin d'élever par son travail, ses petits enfans qui dorment auprès d'elle. Ce seront là mes voisins et mes hôtes. Je m'annonce auprès d'eux comme un passant, comme un étranger qui cherche un pied-à-terre dans le quartier. Je les prie de me céder une portion de leur logement, ou de m'en trouver un dans leur voisinage. J'offre un bon prix, et m'y voilà installé.

Je me garde bien, pour m'attacher ces honnêtes gens, de leur donner de l'argent et de leur faire l'aumône; j'ai des moyens plus honnêtes de gagner leur amitié. Je les charge de me faire des provisions superflues, dont ils profitent: je donne des récompenses à leurs enfans pour de petits services qu'ils m'ont rendus; je mène, un jour de fête, toute la famille à la campagne, dîner sur l'herbe; le père et la mère retournent le soir à la ville, bien restaurés, et chargés de vivres pour le reste de la semaine. A l'entrée de l'hiver, je couvre leurs enfans d'étoffes de laine, et leurs petits membres réchauffés me bénissent, parce que mes bienfaits superbes n'ont point glacé leur cœur. C'est le parrain de leur petit frère qui leur a fait présent de leurs habits. Moins on étreint les liens de la reconnoissance, plus ils se resserrent.

Je n'ai pas seulement le plaisir de faire du bien, et de le faire à propos; j'ai encore celui de m'amuser et de m'instruire. Nous admirons dans nos livres les travaux des artisans, mais nos livres nous enlèvent la moitié de notre plaisir et de la reconnoissance que nous leur devons. Ils nous séparent du peuple, et ils nous trompent en nous montrant les arts avec un grand appareil et de fausses lumiéres, comme des sujets de théâtre et de

lanterne magique. D'ailleurs, il y a plus de savoir dans la tête d'un artisan que dans son art, et plus d'intelligence dans ses mains, que dans le langage de l'écrivain qui le traduit. Les objets portent avec eux leur expression: *Rem verba sequuntur.* L'homme du peuple a de plus une manière d'observer et de sentir, qui n'est pas indifférente. Tandis que le philosophe s'élève tant qu'il peut dans les nues, il se tient lui au fond de la vallée, et il voit bien d'autres perspectives dans le monde. Le malheur le forme à la longue, tout comme un autre. Son langage s'épure avec les années; et j'ai remarqué souvent qu'il y avoit fort peu de différence en justesse, en clarté et en simplicité, des expressions d'un vieux paysan à celles d'un vieux courtisan. Le temps efface de leurs langages et de leurs mœurs, la rusticité et la finesse que la société y avoit introduites. La vieillesse, comme l'enfance, met tous les hommes de niveau, et les rend à la nature.

Mais, dira-t-on, n'y a-t-il rien à craindre dans ce genre de vie? Puissé-je trouver le terme de mes jours dans l'exercice de la vertu!

A translation from the French of J. G. Zimmerman, the celebrated author of "The advantages and disadvantages of solitude."

ON THE ENGLISH NATION.

HAIL, Great Britain! England, fortunate Isle, I congratulate thee! Thou art the abode of joy and pleasure. Thou art the great nursery of heroes, the seat of attractions, the residence of Minerva, and the dominion of Mars. Commerce, the arts and sciences flourish in thy bosom. Delightful abundance reigns in the midst of thee; and thy fields are covered with golden harvests.

In every science, and almost in every art, the English have attained as high a degree of perfection

as any other nation; and the kind reception they give strangers, as well at home as abroad, is an incontestible proof that they have a perfect knowledge of the superiority which they enjoy. If we observe the honours which they bestow upon their fellow-citizens in any situation, we may easily judge how sensible and proud they are of their merits.

There is no country in the world which has less regard to personal distinction arising from birth, rank, circumstances, or other advantages. In Germany, if a stranger is introduced, the first enquiry which is made is this, "Is he a nobleman?" In Holland, "Is he rich?" But in England, "Is he a man of reputation?" In England, the burial-place of kings is also the tomb of men of genius: The ashes of an actress, which, in France, are refused christian burial, are deposited near those of the chiefs of the state.

In this Seminary of great men, the immortal Newton enjoyed, during his life, honours the most extraordinary; and after his death, he was interred with royal pomp and magnificence among kings and men of genius. The honours which, in England, are generally conferred on talents, have, in all ages, impelled the first nobility (to which rank many have raised themselves by means of commerce,) to add, as well as they were able, the palm of science and literature to their escutcheons.

If the English are the most enlightened people, it is solely because they possess a greater degree of freedom than any other nation. Aided by this spirit of liberty, and the inestimable advantages resulting from their constitution, (of which liberty is the basis,) they apply themselves with ardour to the study of the sciences, reflect profoundly on the interests of nations, and employ all their time in deliberating on important objects, or in executing great designs.

May the inhabitants of the British Isles always highly respect their admirable and excellent constitution!

May they always form a proper estimate of its real value! May they never forget that, under it, they enjoy, not only perfect tranquility, but every kind of protection and security, both to their persons and property, such as is unknown in any other part of the world! J. VINT.

The Rev. W. Vint's Academy, Idle.

Animi Immortalitas.

EGO vestros patres, P. Scipio, tuque, C. Læli, viros clarissimos, mihique amicissimos, vivere arbitror: et eam quidem vitam, quæ est sola vita nominanda. Nam, dum sumus in his inclusi compagibus corporis, munere quodam necessitatis, et gravi opere perfungimur. Est enim animus cælestis ex altissimo domicilio depressus, et quasi demersus in terram, locum divinæ naturæ æternitatique contrarium. Sed credo, Deos immortales sparsisse animos in corpora humana, ut essent, qui terras tuerentur, quique cælestium ordinem contemplantes, imitarentur eum vitæ modo atque constantia. Nec me solum ratio ac disputatio impulit, ut ita crederem: sed nobilitas etiam summorum philosophorum, et auctoritas. Audiebam Pythagoram, Pythagoreosque, incolas pæne nostros, qui essent Italici Philosophi quondam nominati, nunquam dubitasse, quin ex universa mente divina delibatos animos haberemus. Demonstrabantur mihi præterea quæ Socrates supremo vitæ die de immortalitate animorum disseruisset, is, qui esset omnium sapientissimus oraculo Apollinis judicatus. Quid multa? sic mihi persuasi, sic sentio, cum tanta celeritas animorum sit, tanta memoria præteritorum, futurorumque prudentia, tot artes, tantæ scientiæ, tot inventa, non posse eam naturam, quæ res eas contineat, esse mortalem. Cumque semper agitetur animus, nec principium motus habeat, quia se ipse moveat; ne finem quidem habiturum esse motus, quia nunquam se ipse sit relicturus. Et, cum simplex animi natura esset,

neque haberet in se quidquam admistum dispar sui, atque dissimile, non posse eum dividi: quod si non possit, non posse interire.

Apud Xenophontem autem moriens Cyrus major, hæc dicit: "Nolite arbitrari, ô mihi carissimi filii, "me, cum a vobis discessero, nusquam, aut nullam fore. "Nec enim, dum eram vobiscum, animum meum vi- "debatis: sed eum esse in hoc corpore, ex iis rebus, "quas gerebam, intelligebatis. Eundem igitur esse "creditote, etiam si nullum videbitis. Nec vero cla- "rorum virorum post mortem honores permanerent, si "nihil eorum ipsorum animi efficerent, quo diutius me- "moriam sui teneremus. Mihi quidem nunquam per- "suaderi potuit, animos, dum in corporibus essent "mortalibus, vivere: cum exissent ex iis, emori: nec "vero tum animum esse insipientem, cum ex insipienti "corpore evasisset: sed cum omni admixtione corporis "liberatus, purus et integer esse cæpisset, tum esse "sapientem. Atque etiam, cum hominis natura morte "dissolvitur, cæterarum rerum perspicuum est, quo "quæque discedant: abeunt enim illuc omnia, unde orta "sunt: animus autem solus, nec cum adest, nec cum "discedit, apparet. Jam vero videtis, nihil esse morti "tam simile, quam somnum. Atque dormientium "animi maxime declarant divinitatem suam. Multa "enim, cum remissi et liberi sunt, futura prospiciunt. "Ex quo intelligitur, quales futuri sint, cum se plane "corporis vinculis relaxaverint."

An censes (ut de me ipso aliquid more senum glorier,) me tantos labores diurnos nocturnosque domi militiæque suscepturum fuisse, si iisdem finibus gloriam meam, quibus vitam, essem terminaturus? nonne melius multo fuisset, otiosam ætatem, et quietam, sine ullo labore aut contentione traducere? sed nescio quomodo animus erigens se, posteritatem semper ita prospiciebat, quasi, cum excessisset è vita, tum denique victurus esset. Quod quidem ni ita se haberet, ut animi immortales essent, haud optimi cujusque animus maxime ad immor-

talem gloriam niteretur. Quid quod sapientissimus quisque æquissimo animo moritur, stultissimus iniquissimo? Nonne vobis videtur animus is, qui plus cernat et longius, videre, se ad meliora proficisci; ille autem, cujus obtusior sit acies, non videre? Equidem efferor studio patres vestros, quos colui et dilexi, videndi; neque vero eos solum convenire aveo, quos ipse cognovi; sed illos etiam, de quibus audivi, et legi, et ipse conscripsi. Quo quidem me pròficiscentem haud sane quis facile retraxerit, neque tanquam Peliam recoxerit; quod si quis Deus mihi largiatur, ut ex hac ætate repuerascam, et in cunis vagiam, valde recusem: nec vero velim, quasi decurso spatio, ad carceres a calce revocari. Quid enim habet vita commodi? quid non potius laboris? sed habeat sane: habet certe tamen, aut satietatem, aut modum. Non lubet enim mihi deplorare vitam, quod multi, et ii docti, sæpe fecerunt. Neque me vixisse pœnitet: quoniam ita vixi, ut non frustra me natum existimem: et ex hac vita ita discedo, tanquam ex hospitio, non tanquam ex domo. Commorandi enim natura diversorium nobis, non habitandi locum dedit. O præclarum diem, cum ad illud divinum animorum concilium cœtumque proficiscar, cumque ex hac turba et colluvione discedam! proficiscar enim non ad eos solum viros, de quibus ante dixi; sed etiam ad Catonem meum, quo nemo vir melior natus est, nemo pietate præstantior: cujus à me corpus crematum est; quod contra decuit ab illo meum. Animus vero non me deserens, sed respectans, in ea profecto loca discessit, quo mihi ipsi cernebat esse veniendum. Quem ego meum casum fortiter ferre visus sum: non quod ego animo ferrem: sed me ipse consolabar, existimans, non longinquum inter nos digressum et discessum fore. His rebus, Scipio, (id enim te cum Lælio admirari solere mihi dixisti,) levis est senectus, nec solum non molesta, sed etiam jucunda. Quod si in hoc erro, quod animos hominum immortales esse credam, lubenter erro: nec mihi hunc errorem, quo delector, dum vivo, extorqueri

volo. Sin mortuus, (ut quidam minuti philosophi censent,) nihil sentiam; non vereor, ne hunc errorem meum mortui philosophi irrideant. Quod si non sumus immortales futuri, tamen extingui homini suo tempore optabile est. Nam habet natura, ut aliarum omnium rerum, sic vivendi modum. Senectus autem, peractio ætatis est, tanquam fabulæ: cujus defatigationem fugere debemus, præsertim adjuncta satietate. Habui hæc, de senectute quæ dicerem: ad quam utinam perveniatis, ut ea, quæ ex me audistis, re experti probare possitis.

A translation from the Latin of Caspar Barlæus.

TO HUGH GROTIUS,

ON HIS DELIVERANCE FROM CONFINEMENT.

Long had brave Grotius been detain'd
In bolted cells, where darkness reign'd;
His soul all further durance spurn'd,
And with desire of freedom burn'd:
But,—Ah! a pris'ner's hapless plight!—
Can he e'er hope to see the light?
He waited, hesitated, plann'd,
And trac'd his course with trembling hand,
God's providential goodness prov'd,
And saw again the light he lov'd.

But first a chest of solid boards
A welcome asylum affords.
Yes, he, of vast capacious mind,
Was in a wooden chest confin'd;
Although that mind would oft essay
Beyond this universe to stray,
And in supernal raptures rove
Where meaner souls durst scarcely move.

A freed-man, and yet bound, he sails;
And, as or hope or fear prevails,

Feels in success his pleasures rise,
Or, thoughtful, broods in sad surmise.
These were his thoughts,—" This great design
" Will make the wonted day-light mine;
" Or else, unfortunate once more,
" I shall this enterprize deplore."

Man! Why astonishment display?
No freed-man treads a common way;
But by the wond'ring world admir'd,
His keepers left, pursuers tir'd,
He dares his liberty confide
On schemes and projects hitherto untried.

Leeds. S. S.

VERSES ADDRESSED TO MADAM GROTIUS,

THE DELIVERER OF HER HUSBAND.

By Caspar Barlæus

GENTLE virgins, now learn what a wife can achieve,
What pleasing inventions her mind can conceive,
How heroic a Belgian's piety shines
When the good of another is in her designs.

See! a wife, quite enchanted, a captive remains,
While she blesses her husband for leaving his chains;
The success of her artifice gladdens her heart,
And her efforts delightful emotions impart.

On her shoulders she did not her husband convey;
But, to free him, she chose an ingenious way,—
For she called him her treasure, and anxiously hid
This great treasure in planks, and closed the lid.

On poles it was carry'd;—the eventful machine
Was securely contriv'd its great inmate to screen,
And within this small mansion his body to coop,
Required that Grotius should dwindle and stoop.

Was the like ever heard?—What this chest might inclose
Was unknown to the bearers, his vigilant foes;
And his guards, quite unconscious, assistance afford
In deliv'ring the man whom they fear'd and abhorr'd.

Unto Breda, by water, the passage is short,
Where his friends are in waiting their man to transport;
When, in secret, the chest of its load they discharge,
And his eminence, Grotius, once more is at large.

Of your females heroic, past ages, keep peace;
Your vantage is lost, let your triumphing cease;
From the wives of your heroes, in courage outdone,
Fair Grotia the palm of affection hath won.

Leeds. S. S.

We received other good themes and translations from Miss M. F. S. and Masters J. Watson, H. Allen, B. Croft Goodison, S. W. Popham, T. Morton, W. Clarke, and George Wells.

Dulces ante omnia Musæ. VIRG.

THE INWARD CONFLICT.

FROM THE FRENCH OF RACINE.

My God! what strife distracts my breast!
By two contending men possess'd!
One strives my heart from sin to free,
And bids me rest my faith on thee:
The other,—to thy holy will a foe,—
Makes me rebel, and sin against thy law.

The one, celestial, full of love,
Directs my thoughts to things above;
Bids me my mind on heav'n to set,
And ev'ry vain pursuit forget:
The other strives my rising to prevent,
And, by his weight, to earth still keeps me bent.

When shall these inward strivings cease?
Ah! whither shall I fly for peace?
I wish—sincerely wish—in vain;
Alas! my wish I ne'er obtain:
The good I love, I cannot do;—my fate
Prompts me to do the evil that I hate.

O Grace! O soul-reviving ray!
Do thou these inward feuds allay:
By thy strong arm the tempest quell,
And from my breast the foe expel:
Let me experience, Lord, thy gentle reign,
Who long the slave of sin and death have been.

Leeds. T. G.

VERSES

Supposed to have been written, by a Person confined in one of the Cells of the Inquisition.

WITHIN these damp and dreary walls,
Where tyranny the heart appals,
And fills with secret dread;
Where noisy chains, fell mis'ry's sound,
Through all these dreary rooms rebound,
I make my dismal bed.

Chain'd in a Dungeon cold and raw,
With Pallet of the worst of straw
On which to lay my head;
Depriv'd of every well-known face,
That might this melancholy chase
And lend me kindly aid;

The prey of calumny and scorn,
I sigh, a poor, distrest, forlorn,
Unhappy, outcast man;
Exil'd the light, immers'd in gloom,
I pace my dark and cheerless room,
Unfriended, sick and wan.

Stern sickness, which confinement brings,
Transfixes with her thousand stings
My feeble, aching frame.
But no kind hand with friendly aid
Uprears my faint and drooping head,
Or cheers life's lambent flame.

No gentle Howard opes these doors,
Or treads these damp, unwholesome floors,
To dry the suff'rer's eyes;
To ease the load which anguish bears;
To weep when sorrow sits in tears,
Or vents her plaints in sighs.

No friendly feet must enter here,
The glimm'ring flame of life to cheer,
Or ease the suff'rer's load:
For superstition bars the gates,
And all the kindness execrates
Which reaches my abode.

No windows to admit the light,
And chase away the murky night,
Except a narrow hole;
This from my cell directly strays,
Attenuates the solar blaze
Which through it seldom stole.

Sometimes, indeed, the orb of day
Darts down a feebly-lengthen'd ray
To light the dreary gloom;
But soon the blessing speeds away,
And darkness takes the place of day
Or twilight, in my room.

Better condemn'd to drag my chains
In slav'ry, on the sun-burnt plains
Of India's sultry zone;
Than *here* be doom'd, without a friend,
To linger, till my being end,
Unnotic'd and alone.

But why this sudden burst of grief?
Why droops my soul if no relief
Of earthly kind can come?
Why wish my days a span in length?
Or why feel pleas'd, when wither'd strength
Proclaims a speedy doom?

Because no *earthly friend* is nigh,
To ease the long and labour'd sigh,
Is GOD departed too?
Has He withdrawn his kindly balm,
Or into tempest wrought the calm
That once my spirit knew?

Leeds, May 10th, 1815. R. B. HOLMES.

VERSES,

ILLUSTRATIVE OF THE MOTTO ON THE MEDAL OF THE ROYAL HUMANE SOCIETY, LONDON.

A genius is represented on this medal, holding, at a small distance from his lips, a flambeau almost extinguished; and is solicitously endeavouring with all his powers to blow up, and raise to a flame, the spark which is on the point of expiring. Its legend—Lateat scintillula forsan, *("Perhaps a very small spark may remain concealed,")—is highly appropriate and full of signification.*

THAT youthful throng their limbs are plying
In the soft and yielding wave;
Each is with the other vying
Which the current's verge shall brave.

Oh! that boy, too much advent'ring,
Quits the shallows near the shore!
The current takes him;—he is ent'ring
On his grave, to rise no more.

See, he beckons for assistance!
How he struggles!—Ah! he'll die!
Proper help is at a distance,
Fly to call yon reapers, fly!

Hastes a boatman within hearing
Ere the youth again doth sink;
Brave!—His arms the lad are bearing
From the fatal river's brink.

Take him to the nearest dwelling,
Place him to be warm and dry;
Soften down this livid swelling;
Gentle remedies apply.

Chafe him;—dead tho' in appearance,
Some vitality remains;
We shall raise, by perseverance,
Circulation in his veins.

. . . . . .

Cease, O woman, cease thy wailing!
Thy son's eyes are clos'd in night;
All our pains are unavailing,
We no spark of life excite.

"Hear his mother, friends and neighbours,
"Feel a childless widow's pains;
"Never intermit these labours
"While a hope of life remains.

"Most forlorn is my condition,
"Earth's best joys I must forego;
"Rapid is this sad transition
"From extreme of weal to woe."

Seeing thee thus sore deserted
Prompts to try what can be done;
Ev'ry nerve shall be exerted,
We will bring to life thy son.

"Sparks of life are yet remaining
"All these doubling folds beneath:
"Oh! delay not! Quick! Be gaining
"On what will not let him breathe!

" All his latent heat elicit,
" Ere his last faint sigh expire;
" Let his lab'ring breast dismiss it;——
" Oh to view him once respire!"

. . . . . .
. . . . . .

Here begins a palpitation
Slightly ruffling all the skin;
This small muscle's agitation
Is a sign of life within.

Look again,—his flesh is wincing,
Slowly move his eye-balls round;
Still he moves—a proof convincing
Life and warmth again are found.

Tardily his blood is flowing,
In its once-accustom'd course;
His extremities are glowing,
And his sighs increase in force.

Either eye with pleasure beaming,
Saw you how he faintly smil'd?
'Tis to him a kind of dreaming——
Woman, take again thy child.

" Great Restorer of his being,
" Blessed be thy gracious name!
" May my child thy goodness seeing,
" In his life thy praise proclaim.

" Thank you too, whose persevering
" Tended to this blest result:
" You a widow have been cheering,
" You have made her heart exult."

Pow'rful is a mother's feeling;——
Though she seem of hope bereft,
She believes, her fears concealing,
Still a spark of life is left.

Leeds. MELPOMENE.

On the difference between experience and wisdom.

ALTHOUGH we acquire wisdom by experience alone, it does not therefore follow that they are the wisest who are the most experienced, or who retain a great deal in their memory; but I affirm, that they are capable of becoming wise, and that experience is the medium which leads to wisdom. For experience, considered merely by itself, is science, and not wisdom; it is the threshold and entrance, by which wisdom may be approached. He is only in the first degree of wisdom, who is possessed of scientific knowledge, and is merely skilled in experiment; for such a person is only acquainted with what is posterior, and is ignorant of what is prior; thus his wisdom does not extend beyond the organs of the senses, and is unconnected with reason; when nevertheless true wisdom embraces both.

In the state of ignorance in which we live, experiment is a kind of phantom, a mere effigy which simulates the appearance of wisdom. At this day they are reputed the most wise who are the most experienced, or most versed in experiment, by making a display of which they are immediately regarded as persons of acute judgment and refined perception; and the more so, if they are possessed of eloquence and an harmonious series and connection of language; and still more so, if they know how to tickle and captivate the ears of their auditors, by a sweetness and melody of voice and accent. But they alone arrive at the goal of true wisdom, who not only possess the greatest store of experience, but have also their organs so formed and disposed, from the senses even to the soul, by means of exercise, and so well and closely connected and arranged, that they can adduce from their treasures of experience, whenever there is occasion, such instances, and such only, as are adapted to the present case and opportunity; by the similitude, analysis, and comparison of which, they are enabled to reason distinctly, and to

arrive even at the causes of the subject of inquiry, or at the things antecedent and prior to it, by a chain of argument.

But experience, taken by itself, as was said before, is not wisdom. A painter who is possessed of colours and dyes, and can draw lines with them, is not therefore master of his profession; nor is a mere manufacturer of instruments capable, on that account, of touching the strings of the harp with harmony and skill. He who possesses an ostentatious number of books, is not necessarily a man of learning; nor does he, for that reason, deserve the laurel crown; for his wit may very probably be of the dullest and grossest kind. Or, to consider the matter nearer and more interiorly, the historian who has turned over a multitude of books, and has learned from them the fates and vicissitudes of former ages, and the lives and exploits of all the heroes, is not on that account wise, and worthy of being raised to the chair of state; that is, he is not, from that circumstance alone, an able member of the commonwealth, and more deserving than others to be seated at the helm. He ought to have the events and transactions of former times so arranged, by means of his organs and the various chambers of his memory, as to be able, on every occurrence, to refer to such historical circumstances as are most similar and analogous to the case in hand, and these, and no others, ought, as if spontaneously, to offer themselves to his recollection. Nor is he even then wise, unless he has previously penetrated, by means of a rational philosophy, into the causes and principles of things, in order that he may afterwards be able to reason respecting present subjects, from causes and principles, or from reason *a priori*, and to form more certain conclusions by means of a connected chain of inferences; and, having his counsels derived from such a source, that he may always be able, by the timely adoption of proper measures, to provide for the welfare of the state.

Hence it follows that he who retains all the natural experience of the world laid up in the storehouse of memory, is not on that account a philosopher, and capable of knowing the causes of things, and of reasoning *a priori*; to do which, he must know how to digest all things analytically, by means of geometry and rational philosophy; and must possess the faculty of reasoning philosophically, which consists in the situation and figure of the organs, connected with the rational faculty by continual cultivation and use.

It is thus that a man may first become a philosopher, may be enabled to penetrate into the causes of things, and afterwards, from causes, to speak by means of experience or experiment. PRINCIPIA.

ON MUSIC.

(Continued.)

JUDGMENT in music is shewn by adapting such harmonious accompaniments to the melody as may give it a variety, without destroying its simplicity; in the preparation and resolution of discords; and in the artful transition from one key to another. Taste, in a performer, consists in a knowledge of the composer's design, and expressing it in a spirited and pathetic manner, without any view of shewing the dexterity of his own execution. But though all these circumstances of composition and performance, should concur in a piece of music; yet it must always fail in affecting the passions, unless its meaning and direction be ascertained by adapting it to sentiment and pathetic composition. It exerts its greatest powers, when used as an assistant to poetry; hence the great superiority of vocal to instrumental music. The human voice is capable of more justness, and a more delicate musical expression, than any instrument whatever; the perfection of an instrument depending on its nearest approach to it.

Vocal music is much confined by the language in which it is performed. The harmony and sweetness of the Greek and Italian languages give them great advantages over the English and French, which are harsh, unmusical, and full of consonants; and this, among other inconveniences, occasions perpetual sacrifices of the quantity to the modulation. This is one great cause of the slightness and want of variety of the Freneh music, which they in vain endeavour to cover and supply by laboured and complex accompaniments.

As vocal music is the first and most natural music of every country, it is reasonable to expect some analogy between it and the poetry of the country, to which it is always adapted.—The great superiority of the Scotch songs to the English, may. in a great measure, be accounted for from this principle. The Scotch songs are simple and tender, full of strokes of nature and passion: So is their music. Most of the English songs abound in quaint and childish conceits. They all aim at wit, and sometimes attain it; but music has no expression for wit, and the music of their songs is therefore flat and insipid, and so little esteemed by the English themselves, that it is in a perpetual fluctuation, and has never had any characteristic stile.

On the other hand, England has produced many admirable composers of church music. Their great attachment to counterpoint has often led them into a wrong track; in other respects, they have shewn both genius and taste. Religion indeed opens the amplest field for musical, as well as poetical genius; it produces almost all the variety of subjects which music can express, the sublime, the joyous, the cheerful, the serene, the devout, the plaintive, the sorrowful, It likewise warms the heart with that enthusiasm so peculiarly necessary in all works of genius. Accordingly the finest compositions in music which we have, are in the church stile. Handel far advanced in life, when his constitution and spirits seemed nearly exhausted, was so roused by this

subject, that he exhibited proofs of extent and sublimity of genius, in his Messiah, superior to any he had shewed in the most vigorous and happy period of his life. We have another instance of the same kind in Marcello, a noble Venetian, who set the first fifty psalms to music. In this work he has united the simplicity and pathos of the ancient music with the grace and variety of the modern. In compliance with the taste of the times, he has sometimes been forced to leave that simplicity of stile which he loved and admired; but by doing so, he has enriched the art with a variety of the most expressive and unusual harmonies.

The great object in vocal music is to make the music expressive of the sentiment. How little this is usually regarded, appears by the practice of singing all the parts of a song to the same music, though the sentiments and passions to be expressed be ever so different. If the music has any character at all, this is a manifest violation of taste and common sense, as it is obvious every different sentiment and passion should be expressed in a stile peculiarly suited to itself. But the most common blunder in composers, who aim at expression, is their mistaking imitation for it.

Frugality in individuals the source of prosperity to nations.

Some time since a venerable old man, who had filled all the important municipal offices, in one of the chief towns in Holland, with honour and reputation, and had gained great riches without reproach, began to entertain some thoughts of retiring for the rest of his days to his country-seat. In order to take leave of his friends and acquaintance in a handsome manner, he invited the young and old of both sexes, and of the best fashion in the place, to an entertainment at his own house. They met together with great expectations, but, to their no small surprize, they saw a long oak table, scarcely

covered with a scanty blue cloth, on which were alternately placed platters of butter-milk, sour curd, pickled herrings, and cheese. The rest of the cheer was made up with butter and rye-bread, and cans of small-beer were at hand, for those who had a mind to drink. Trenchers served instead of plates, and not a single servant attended. The company secretly cursed the old man's humour: but out of regard to his great age, and to his still greater merit, they bridled their resentment, and seemed content with their homely fare.

The old gentleman seeing the joke take, was unwilling to carry it too far; and, at a signal given, two clean boorish maids, in their country garb, shifted the table, and brought in the second course; the blue cloth was changed for white linen, the trenchers for pewter plates, the rye-bread to household-brown, the small into strong-beer, and the mean food into good salted beef and boiled fish. The guests grew better pleased, and the master of the feast more pressing in his invitations.

After he had given them time to taste of the second course, a third was served up by a *maitre d'hotel* in form, followed by half a dozen powdered lacquies in gawdy liveries; the most beautiful flowered damask was spread on a sumptuous mahogany table; the richest plate and most curious china adorned the sideboard; whilst a profusion of soups, olios, tame and wild fowl, fricassees, ragouts, in a word, all that the art of a modern French cook could produce, ranged in a well-disposed, judicious order, seemed to court the taste, and renew the appetite of the whole company. To these were added strong-bodied Chateau-Margen, generous Burgundy, sparkling Champaign; in short, a choice of the best wines which commerce can procure to a trading country; and, that nothing might be wanting which could please the senses, as soon as a sumptuous desert was brought in, a melodious concert of an agreeable variety of instruments was heard in the next

room; healths went round, mirth increased, and the old man seeing that nothing but his departure and that of the gravest of the company was waited for, to give a loose to joy and pleasure, rose up, and made the following speech:

"Ladies and gentlemen, I heartily thank you for this "favour; it is time for one of my age to withdraw; but "I hope those who are disposed for dancing will accept "of a ball, which I have ordered to be prepared for "them. Before the fiddles strike up, give me leave to "make a short reflexion on the entertainment, which "otherwise would appear whimsical, and the effect of "humour: it may serve to give an idea of our common-"wealth.

"By living after that penurious manner, exhibited in "the first course, our ancestors raised their infant state, "and acquired liberty, wealth and power.

"These were preserved by our fathers, who lived in "that handsome, but plain way, which you have seen in "the second course.

"But if an old man may be permitted, before he "leaves you whom he dearly loves, to speak his thoughts "freely, I am indeed afraid that the extravagant plen-"ty which you have observed in the last course, will, "if we go on, deprive us of those advantages which "our ancestors have earned by the sweat of their brows; "and which our fathers, by their industry and good "management, have transmitted to us.

"Young people, I advise you to be merry this even-"ing, but to think seriously to-morrow on what I have "been telling you to-day. Good night." J. S. E.

To the Editor of the Leeds Correspondent.

SIR,—You appear to be a lover of practical morality; and this consideration is a great encouragement to a plain man, of homely habits, in addressing you. I am indebted to an uncle for much of my knowledge of

men and things. When I was a boy, he often took me with him to Leeds market. We commonly walked thither on foot; and, after having sold or left his cloth, we had the satisfaction of riding home together on his bay gelding. I took great delight in his company; for he drew something of entertainment or instruction from every occurrence. He taught me to exercise my powers of discrimination, and early to distinguish between pretension and reality, truth and the semblance of it. And since I find it to be a secret with which it would be well if many were acquainted who appreciate its value too late; I have in this manner revealed it to you for the good of the community.

When a young man is about to be introduced into society, great attention ought to be paid to his natural disposition. If he possess a native bias to distrust or jealousy of mankind, it will be needful for him to be introduced to the bright side of the picture; when it should be demonstrated to him,—though vices and abominations prevail among men, yet that many of them have acquired great virtues, and manifest them in their practice. If, however, he exhibit a generous, open, and unsuspecting temper, the vicious gloom which overshadows the greater portion of our species should be pointed out to him. This moral darkness ought not to be contemplated by him with trembling at a distance, though that will always be the sensation experienced by every well-instructed youth at the first sight of the hideous spectacle. But he should be taught to approach, and inspect it nearly and with interest, as being a representation of part of the dense atmosphere with which, in future life, he will occasionally be surrounded. And he should be told, that he need not stumble, in the most dreary and difficult part of his journey, if he employ for his light and guidance the lanthorn with which he is furnished by his Great Benefactor.

Being free in my disposition, the latter was the method which my intelligent uncle observed in his treatment of me. He omitted no opportunity of pourtraying, wi

a gentle yet faithful hand, the motives of men, as seen in their first movements and demonstrated in the catastrophe of their deeds. Having an insight almost intuitive into characters, I scarcely knew him at any time to be mistaken. If in misapprehension he had said any thing to the prejudice of another, the frankness of his heart impelled him at once to acknowledge the error and to put his own contradiction on it. To his early trials, and long experience in the world, I attribute his attainment of this skill, which was so just as to entitle it, in numerous instances, to a near alliance with the spirit of true prognostication.

The first lesson on the developement of character that I received from him, was an exemplification of this, one of his own sterling proverbs,—*Bright buttons do not of themselves make a good coat, nor are handsome ruffles always attached to a fine shirt.* These expressions may now appear uncouth and irrelevant; but they were then particularly sententious and appropriate, as large metal buttons and splendid ruffles had become all the vogue. The following occurrence gave rise to them:

In one of our excursions to Leeds, we met with a spruce gentleman, who had purchased a valuable farm in our neighbourhood and had come to reside on it.—He had understood that my uncle was well versed in township-affairs, and he was desirous of being instructed by him in some particulars relating to the poor's-rate imposed on his land and premises, the poundage of which had then been but recently increased. In order to succeed, he accosted my uncle with the freedom of an old friend and all the urbanity of a professed courtier. Though he saw us driving the nag along before us loaded with cloth, he began to pester my uncle with invitations to dinner, who civilly informed him that he was directing his course towards Leeds, and could not then on any account be detained. The gentleman renewed his importunities; and perceiving me, an ignorant lad, to be big with amazement at his astonishing volubility, he plied my uncle, (on his weak side, as he

supposed,) with encomiums on me: "Is this your "eldest son, Mr. R.? He is a fine youth. Do you in- "tend him for a clothier?" And thus he ran on, scarcely allowing time for my uncle to say, "John is my nephew, Sir." This was but a new track for his fluency to run along; at the end of which, he began to give some hints of the business which he wished to introduce. When he was told, that we would call on him as we returned in the afternoon, he departed from us, expressing, in the most captivating manner, the amount of his regret, (which, he said, was *very great*,) at not being able to succeed in his application for the honour of our company at dinner.

We pursued our journey, arrived at Leeds, finished our business, and called, as requested, about four in the afternoon, at the gentleman's house, who was again profuse in his compliments. My uncle, a man of few words, knew, by the conversation which he had before held with him, what he wanted, and interrupted his oratory by entering immediately on the subject of consultation. Having satisfied him concerning the particulars which he had mentioned, and answered his enquiries about the usages of neighbouring townships,—he prepared for his departure by taking up his hat, which he had not suffered to be removed from the top of his staff held upright in his hand, and attempted to march across the room to the door-way. In completing his intention he was hindered by the genteel officiousness of his host, who begged him to make a longer stay with him, and to gratify him so far as to partake of some refreshment. In the act of persuasion he laid his tapering fingers on my uncle's broad shoulders; and in this compulsive posture exerted his finest rhetoric to procure from him the favour of his further presence. Giving his shoulders a natural shrug, and his body a motion forward, he freed himself from this elegant obstruction, and tartly said, "Mr. Flippant, I esteem "a man who has a will of his own, and who is persuad- "ed that he knows, better than another, what has the

"greatest claim on his present attention. I cannot
"accept of your invitation;—and so, good evening!"

I had been much pleased in the morning with the courtesy and affable ease of the man; and had secretly wished that our engagements would have permitted us to stop dinner. I had, however, said nothing about him to my uncle. But now, when so much gentility had been displayed on one side, and so much apparent bluntness on the other, I could not refrain from interrogating my uncle on the reasons of his conduct in declining the gentleman's invitation. 'John, my lad, said he, 'when a man overstrains a matter, his words and 'actions have the appearance of insincerity.—Bright but-'tons do not of themselves make a good coat, nor are 'handsome ruffles always attached to a fine shirt: Nei-'ther at all times give credence to a man because of the 'abundance of his professions. This very man is both 'a fool and a knave. I never knew him till nearly 'four months ago, when, one very rainy day, neighbour 'Homespun and I called at his house, to acquaint him 'with the required statute duty. He would compel 'us to drink a glass of spirits with him; which we did, 'and in compliance with his humour sat, and discours-'ed on many public and private occurrences. While 'we were thus occupied, his servant came and an-'nounced to him that Thomas Todd who farms two of 'his fields, had come to pay his rent; "Oh! Let Thomas "be brought in!"—Thomas came, and was accosted 'with "Well, my brave fellow, Thomas! How is it with "you?"—"Middling, Sir; I have come to pay my "rent."—"What a pity that you have given yourself "the trouble of bringing it in such unpleasant weather! "When you know me better, you will find that I am "not particular as to a few weeks with such a respect-"able man as you. Will you have a glass of spirits?" "As you please, Sir," said Thomas, "I do not know "you right yet; but your servant said, he delivered a "right message when he told me that unless I paid you "my rent during the week, (and this is the last day

" in it,) you would sue me for it." At this Flippant ' looked a little confused, but instantly rejoined in a con-' sequential manner, " My man is not fit to take any " message. What a roguish invention he must have " to utter such a tale! I will take him to task about " this matter. But, good neighbour, you shall stop tea " with me."—Thomas drank off his glass; and courage-' ously resisting his landlord's impertinent assiduities, ' went away. When the door of the room was closed ' after him, with a smile he had the *folly* to display his ' *knavery* before us, who were till then strangers to him, ' by saying, " 'Tis very well;—A simple clown! Did " he imagine that I wanted him to stop?"—On hearing ' this, Homespun and I soon managed to retreat, in ' decent order; though, as simple clowns, we were also ' assailed with the same civilities as those which had ' been bestowed on Thomas. And from that time I have ' resolved to contract no intimacy with such a despica-' ble character. When he is better known, he will be ' universally despised.'

From this circumstance I viewed Mr. Flippant as a hypocritical monster, of which, I hoped for the honour of human nature, the world could not afford many specimens. At that early period of my life it seemed to me ridiculous in the extreme for any man to behave himself with such hollow insincerity, and to take such uncommon pains to appear as a person of a generous disposition, when one fourth of his exertions would have sufficed to make him really so.

As this was one of my first discoveries of human duplicity, it gave me continued and serious uneasiness; and it was long before I could be brought to believe my senses, those channels of intellectual communication, that many as egregious vices, in polished society, often deface, from the soul of man, the fair image of his Creator.—Happy were those days of unsuspecting innocency. It remained for riper years painfully to convince me, that Mr. Flippant was not the only man of that breed, whose *bright buttons are no proof of a good*

coat. I have lived to see the words of my uncle completely verified: Flippant was neglected by every man of principle; and, when he died, was lamented only by his children. His decease caused no mournful break in the ranks of social life, which men are apt to say can never be filled up so well as by the individual who quitted it.

I may reveal other particulars of my secret, on a future occasion; and am in the mean time,

Yeadon, Yours &c.

JOHAN. RUSTICUS.

QUERIES.

(1) *By* T. PILKINGTON, *Haslingden.*

WHAT are the two grand principles or faculties of the human mind?

(2) *By* QUÆSITOR, *Manchester.*

WHAT is the plural of the noun "*valley*?"

(3) *By* Mr. J. BAINES, *Reading.*

DOES snow, when abundant in quantity, fertilize the ground, if it lie a considerable time upon it?

(4) *By the same.*

WHAT is it that causes the bones of several animals to petrify, or to be converted into stone, when they are buried in the earth?

(5) *By* INVESTIGATOR.

How ought all the members of the following perfect sentence to be parsed?—"The Lord—he is good."

(6) *By* W. STRINGER.

IF Oxygen be the acidifying principle, and 85 in 100 parts of water be Oxygen, why has not water an acid taste?

(7) *By the same.*

Why do drops of rain on the leaves of plants assume a globular form?

(8) *By* Mr. F. Charlton, *Kirkheaton.*

Do English tribunals recognize the validity of Gretna-Green marriage's; and are they legally valid between two persons domiciled in England?

ANSWERS TO THE QUERIES

Which were proposed in the Fourth Number of the first Volume.

Answers to all the Querics were received from Sir Dan. De Grandew, Messrs. Baines, and Job Shainen. We were also favoured with answers to several of them, from Messrs. Charlton, Pilkington, and Brooke; from Anasthimenes, Investigator, Tippoo, and Quæsitor.

(1) On the germination of a potatoe.

By Sir Daniel de Grandew, *Normanby.*

After perusing this query, I had occasion to take up some potatoes in my garden, September 23d, 1815: and I found that most of them had sprouts on them, from 1 inch to 1½ in length; and one of them had actually a new potatoe, about the size of a large pea, at the end of it. And in January or the beginning of February, 1813, I ate of some potatoes, which had grown in a dunghill by accident: they were, in general, about an inch and a half in diameter.—It appears, therefore, that the potatoe is disposed to sprout in any season, if there exists a due degree of natural or artificial warmth to support the vegetation. But in Autumn, though the atmosphere may be of the same temperature as in spring when germination commences, yet the succeeding cold weather stops its progress, and

renders the mucilaginous and saccharine matter quiescent. In this manner a nutritive article of food is preserved, for culinary purposes, or to assist in the propagation of a new plant in early spring.

But if we admit of the irritability and sensibility of plants, we might argue that the periods at which they thrive, are greatly influenced by their acquired habits, and by having their sensibility of heat encreased in spring, in consequence of their being exposed to the cold of winter. In a similar manner, the limbs of animals acquire a greater sensibility of heat, after they have been exposed to cold. Thus, as the hands glow with warmth, on entering the house after they have been immersed in snow, so does the spirit of vegetation acquire additional vigour after the potatoe has endured the rigours of winter. Yet, plausible as this may appear, "We must not suffer ourselves to be deluded by the very extensive application of the word "life", to conceive, in the life of plants, any power similar to that producing the life of animals. In calling forth the vegetable functions, common physical agents alone seem to operate; but in the animal system, these agents are made subservient to a superior principle. There are few philosophers who would be inclined to assert the existence of any thing immaterial in the vegetable economy; though vegetables may possess the means of converting the elements of common matter into organized structures, both by assimilation and reproduction."

By JOB SHAINEN, *London.*

A POTATO will not germinate, when it is put into the earth, unless the soil be moist and contain a sufficient quantity of warmth for promoting vegetation; this, it has been observed, is the case both in spring and in autumn. For if a potatoe be put into the ground ever so early, it will not germinate so as to appear above the surface, until it is called forth by the warmth of the weather. From this it appears, that the principal

cause why a potatoe will not germinate in autumn, is the want of warmth in the earth.

(2) On the effects of paring and burning the soil.

By Sir DE GRANDEW, *Normanby.*

THE learned and ingenious Sir Humphrey Davy observes (see Elements of Agricultural Chemistry, pages 346 and 350) that "the process of burning renders the soil less compact, less tenacious and retentive of moisture; and when properly applied, may convert a matter that was stiff, damp, and in consequence cold, into one powdery, dry, and warm; and much more proper for a bed of vegetable life.—The great objection made by speculative chemists to paring and burning, is, that it destroys vegetable and animal matter, or the manure in the soil; but in cases in which the texture of its earthy ingredients is permanently improved, there is more than a compensation for this temporary disadvantage. And in some soils where there is an excess of inert vegetable matter, the destruction of it must be beneficial; and the carbonaceous matter remaining in the ashes may be more useful to the crop than the vegetable fibre from which it was produced." The learned Dr. (after an account of several experiments) observes that "many obscure causes have been referred to, for the purpose of explaining the effects of paring and burning; but he believes they may be referred entirely to the diminution of the coherence and tenacity of clays, and to the destruction of inert and useless vegetable matter, and its conversion into a manure."—Lastly, he avers that the stiff clays and marls, are improved by burning; but in course sands, or rich soils containing a just mixture of the earths, and in all cases in which the texture is already sufficiently loose, or the organisable matter sufficiently soluble, the process of torrefaction cannot be useful.

All poor siliceous sands must be injured by it; and here practice is found to accord with theory. Mr. Young, in his essay on manures, states, "that he

found burning to injure sand;" and the operation is never performed by good agriculturists upon siliceous sandy soils, after they have once been brought into cultivation.

By Mr. JOB SHAINEN, *London.*

SOILS which owe their sterility to an abundance of vegetable fibres contained in them, such as old pasture-lands, waste commons, moors, &c. may be rendered fertile by means of paring and burning, or by much pulverization and exposure to the air; and also by the agency of newly-made quick lime. In all other kinds of soil, paring and burning is absolutely injurious, and impoverishes it as much as a crop of corn or vegetables: it destroys, in some measure, the power of absorbing and retaining moisture, a principle which is very essential to productiveness; for in supplying a soil with animal and vegetable manure, temporary food only is provided for plants; but when its constitution and texture, with regard to its earthy parts, are brought into the best state possible, it is then capable of attracting a very large portion of vegetable nourishment from the atmosphere. Consequently, whatever tends to destroy the above property of a soil, must be injurious to it.

(8.) Is a burn alleviated by holding it to the fire?

By ANASTHIMENES.

WHEN any part of the human body has been burnt, the pain will be mitigated, by holding it to the fire, though a considerable degree of additional pain will be created while the part is so exposed. Some persons, perhaps, will account for this by *every like drawing its like*, that is, The heat of the fire draws the unsufferable heat in the hand. But, in a burn, the epidermis, or scarf-skin, comes first in contact with the fire, next the *rete mucosum*, (which may be dissolved by maceration in water,) and then the *cutis vera*, or true skin: The surface of this skin abounds with numerous *papillæ*, or small glands, possessing a most exquisite sensation

and thus forming the sense of feeling. When the fire has penetrated this skin, (as in the application of a blistering plaster,) the first action is a stimulating of the vessels of the skin and a collecting of the *serum* and watery part of the vital fluid under the cuticle, which form a blister, and consequently cause great pain. But when the part is just burnt, before the blisters rise,* by holding it to the fire, this fluid is carried off or absorbed by the heat of the fire; therefore the pain will be considerably abated. But then the scarf-skin generally peels off.

(4) What part of speech is "that?"

By GRAMMATICUS.

EVERY language, even the purest, has its own anomalies, words which, in their terminational or other variations, have but few properties in common with the rest of their *genera*. Most of them have something peculiar in the mode of their construction, or in the prominent and arbitrary situation chosen for themselves in a sentence. They cannot be accounted for by any of the rules, which are usually adduced for words nearly resembling them in construction or situation.

The grammarians who meddle with these outlaws of language, may be divided into two classes: The first strive amain to bend every offending expression to a system of rules which they have had great trouble in deducing from the usage and position of other similar words or members of sentences; and to which system, they think, every part of speech, however stiff-necked, ought to submit itself. If it do not remain quiet, and prove flexible and obedient at their requisition, they become rancorous, and declare hostilities and openly wage war against it, proclaiming its defects or excesses to the ends of the earth.

* Perhaps few would try the effect of the fire when the blisters are risen.

The other class being a little better initiated in the philosophy of language, consider many of these words as very valuable in the ranks of speech, and on no account deserving of such insulting treatment. They, therefore, wink at their peculiarities, account for them as exceptions to general rules, and denominate them "anomalies."

If one of the former class were called on to parse the word in question, he would say, that it was an incorrect construction, and *unparseable*, because it did not fall into any partitions, or pigeon-holes, which he had framed for other words in the like circumstances. In order to handle it aright, he knocks it out of the ranks, and substitutes for it, without hesitation, a stiff and circumlocutory method of expression, devoid of the elegance which distinguished its predecessor, and adding nothing to the force of the meaning.

Were one of the latter class called on, he would say, "The impersonal verb "IT IS," with other impersonals, requires in all its moods and tenses the attendance of the word THAT, as a follower, sometimes in a conjunctive or adverbial, and at other times in a pronominal capacity, and that without any breach of propriety. As, *Was it for this* THAT *I rose up early and late took rest? Then it is* THAT *the soul is exalted with enlivening airs. Is it not he* THAT *will guide me?*—But this construction does not obtain, when the word "IT" has the affections of a personal pronoun."

By T. PILKINGTON, *Haslingdon.*

"The word *that* is sometimes a relative, sometimes a demonstrative pronoun, and sometimes a conjunction. It is a relative, when it may be turned into *who* or *which* without destroying the sense: as, "They that (who) reprove us, may be our best friends." It is a demonstrative pronoun when it is followed immediately by a substantive, to which it is either joined or refers, and which it either limits or qualifies: as, "That boy is industrious." It is a conjunction, when it joins sentences

together, and cannot be turned into *who* or *which* without destroying the sense: as, "Take care *that* every day be well employed." (Murray.) In the last case it may possibly be ranked as a conjunction; but, I think, in the paragraph as stated by Mr. Nixon, the sense which the word *that* conveys there, is not properly that of a conjunction, as they in general merely connect words or sentences together: but in the phrase under consideration, the word has an allusion to time, and the sentence may be rendered thus: "It is in the sullen and dark shade of discontent, that *(when, or, at which time)* noxious passions, like venomous animals, breed and prey upon the heart."

It is a distinguishing property of adverbs to have relation to *time, manner, number, place* &c. hence I conclude that the above word is adverbial.

By QUÆSITOR.

IN this sentence the word *that,* whatever authority it may derive from example, is erroneous. It ought to be *in which,* or *where,* referring to the antecedent *shade,* and in Latin it would be rendered by the Ablative case of the relative pronoun.

Dr. Lowth, in the notes to his English Grammar, (page 93.) makes the following quotation: "Breaking "a constitution by the very same errors that many have "been broke (broken) before." 'SWIFT'S *Contests and* 'dissensions, &c.' Upon which he remarks, 'that here 'the relative is employed not only to represent the antecedent noun *the errors,* but likewise the preposition 'by prefixed to it;' and concludes by saying, 'that it 'ought to be *by which,* &c.' The same remark, I presume, is applicable to the case in Question, where the relative is made to represent both the noun *shade* and also the preposition *in.* The sentence therefore ought to be, "It is in the sullen and dark shade of discontent, in "which (or where) noxious passions, like venomous "animals, breed and prey upon the heart."

(5) On the nature of the electric fluid.

By Sir DAN. DE GRANDEW, *Normanby.*

IN Leybourn's Philosophical Repository, (Vol. 2nd Page 94.) Mr. Thomas Squire of Baldock, seems to have proved that light, fire and Electricity are different modifications of one and the same principle. To prove the identity of the electric fluid and fire, he adduces some experiments made with an apparatus constructed at the expence of the board of ordnance and suspended in the Pantheon.—It consisted of a great number of drums, covered with tinfoil, which formed a Cylinder of above 155 feet in length, and more than 16 inches in diameter, and to this vast conductor were occasionally added 4800 yards of wire.—The electric blast from this machine fired gunpowder in the most unfavourable circumstances that can be imagined, namely, when it was drawn off by a sharp point: in which case it has generally less force than in any other. The method of doing this was as follows:—Upon a staff of baked wood, a stem of brass was fixed, which terminated in an iron point at the top. This point was put into the end of a small tube, of Indian-paper, made somewhat in the form of a cartridge, about $1\frac{1}{4}$ inch long, and $\frac{8}{10}$ of an inch in diameter. When the cartridge was filled with common gunpowder, unbruised, a wire communicating with the earth was fastened to the bottom of the brass stem. The charge in the great cylinder being continually kept up by the motion of the wheel, the top of the cartridge was brought very near the drums, so that it frequently even touched the tinfoil with which they were covered. In this situation a small luminous stream was frequently observed between the top of the cartridge and the metal. Sometimes this stream would set fire to the gunpowder, the moment it was applied; at others, it would require half a minute, or more, before it took effect. But this difference in time was supposed to be owing to some small degree of mixture in the powder or paper, which was always unfavour-

able to the experiments.—Tinder was fired much more readily.—"As it therefore appears (says Mr. Tytler) that the electric fluid, when it moves through bodies either with great rapidity or in very great quantity, will set them on fire, it is scarcely disputable that this fluid is the same with the element of fire."

On the other hand, Dr. Van Marum after relating several experiments on the melting of metals, which did not accord with other experiments made by the Academicians of Dijon, concludes that the supposed analogy between these two powerful agents cannot be proved either from the fusion of metals, or of the combustible substances.

By Mr. J. Baines.

It has been imagined that the electric fluid does not set fire to bodies because it is in an ignited state, but because its great velocity excites that intestine motion which raises their temperature. This, I think, is contrary to observation; for when an electrified cloud approaches any thing that is not electrified, either a momentary flash or a train of fire is observed to issue from it. The combustible matter which strikes the ground is of such an intense heat as to burn and consume every thing that is combustible. It has been known to melt metals, calcine the bones of men and animals, throw down buildings, split trees, or tear them up by the roots, pierce the thickest walls, and reduce stones to ashes. It is therefore probable that every electric spark or flash of lightning consists of one or more balls of fire, though its great velocity presents it to the eye under the form of a line.

By Mr. Foster Charlton.

The opinions of Philosophers respecting the nature of the electric fluid have been various; Some supposing it to be precisely the same as fire; others inferring, as it differed in its effects from fire, that it must possess inherent qualities materially different. None of

them however seems to know of what materials its composition consists. Dr. Priestly, from observations on phlogiston and electric fire, concludes, from the similarity to their effects, that they either are exactly the same, or the one contains a great deal of the other. It is evident that the electric fluid is a kind of fire, though Electricians of the greatest celebrity fall short in establishing an identity between it and fire. Like many of the natural phenomena, this seems to remain a mystery; and nothing would be more presumptuous than to attempt to define it from hypothesis. We must rest in incertitude, till experiment teach us its true nature.

(6) The snow on high mountains.

By Mr. J. Baines.

That eternal frost prevails on the tops of the most elevated mountains, even in the torrid zone, has been verified, both by experience and calculation; yet it can scarcely be imagined that they are, at this moment, covered with all the snow that has fallen upon them since the creation of the world. But if we consider a circumstance which comes within our immediate observation, (viz. that a portion of the snow which falls upon a hill, is in a short time, if the weather continues to be frosty, driven from its station, by the force of the wind, into the valley below,) we shall be enabled to account for its diminution, without supposing it to be converted into water.

By Mr. Foster Charlton.

If the snow on the tops of large mountains is not diminished by thaws, the most probable way of its diminution is by the action of the sun. The verity of this supposition is corroborated by the mountains in Scotland and the northern parts of England. Snow has been known to lie for several years on the top

of Cross-fell, a mountain in Cumberland, but is considerably lessened in summer, and totally vanishes in a hot season. The bottoms of mountains whose tops are covered with snow, are generally in a wet state, and particularly when the heat of the sun is intense; hence we may reasonably infer that a great part of the snow on their tops is melted. To imagine it remains entire after its fall, even on the largest mountains, would be highly absurd; for it must suffer a diminution from the warmth of the sun, independent of any thaws, which exist either more or less in almost every climate.

By QUÆSITOR.

THE following extract from that excellent work, the Encyclopædia Britannica, will, perhaps, be deemed a satisfactory answer to this question. See Vol. 7. Page 483. Artic. Frost.

" Even in the polar regions, where it may be thought that the frost must increase beyond measure, there are also natural means for preventing it from running to extremes. The principal cause here is, the mixture of a great quantity of vapours, from the more temperate regions of the globe, with the air in those dreary climates. It is well known that aqueous vapour always flies from a warm to a colder place. For this reason, the vapours raised by the sun in the more temperate regions of the earth, must continually travel Northward and Southward in great quantities. Thus they furnish materials for those immense quantities of snow and ice, which are to be found in the neighbourhood of the poles, and which, we cannot imagine, the weak influence of the sun in those parts, is capable of raising. It is impossible that a quantity of vapour can be mixed with frosty air without communicating a great quantity of heat to it, and thus there are often thaws of considerable duration even in those climates where, from the little influence of the sun, we should suppose the frost would be perpetual,"

ANSWERS

TO THE

QUESTIONS FOR YOUTH,

Proposed in Fourth Number of the first Volume.

(1) *By* Master WM. MARSDEN, *Free Writing-School, Sheffield.*

A grocer gave 30£. for 87¼lb. of tobacco; how must he sell it per ounce to gain 17£ in laying out 99£. 1*s.*?

Solution by Master J. NICHOLSON.

As 30£ : 87¼ *lb* : : 92£. 1*s* : $\frac{691369}{2400}$ *lb* the quantity bought for 99£. 1*s.* which must be sold for 116£. 1*s.* Therefore, As $\frac{691369}{2400}$lb : 116£. 1*s.* : : $\frac{1}{16}$lb. : $\frac{34815}{1382738}$£. $= 6\frac{29586}{691369}$ *d.* the required price.

Another, by Master W. SHEPHERD.

Here 87¼lb. : 30£. : : 1*oz.* : $5\frac{55}{349}$ *d.* the price he gave for 1 ounce; then, 99£.1*s.*:116£.1*s.*:: $5\frac{55}{349}$ *d.* : $6\frac{29586}{691369}$ *d.* the price required.

(2) *By* Master J. HARGREAVE, *Mr. Nesbit's Academy.*

Required the area of a walk made round a circular bath, whose diameter is 12 feet; the breadth of the walk being every where 4 feet.

Solution by Masters B. C. GOODISON and H. ALLEN.

First $12^2 \times .7854 = 113.0976$ the area of the bath, and $20^2 \times .7854 = 314.16$ = that of the walk and bath; consequently 201.0624 = the area of the walk.

The same by Masters J. Robson and Robert Bell.

Here $(20 + 12) \times (20 - 12) \times .7854 = 201.0624$ feet the required area.

(3) *By* Master R. S. Cotton, *Mr. Reynard's Mathematical School, Reading.*

Three persons, A, B, and C, set up in trade together, but, after having sustained several losses, agree to dissolve partnership; when it was found that A and C's share exceeded B's by 250£., A and B's exceeded C's by 200£., and A's was equal to the sum of the other two *minus* 10£. Required the share of each.

Solution by Master J. Vint.

Let x, y, and z represent the shares of A, B, and C respectively. Then we have

1st Eq. $x + z = y + 250$ 2nd $x + y = z + 200$

2nd $x + y = z + 200$ 3rd $x = y + z - 10$

$2x = 450$ by add. $2y = 210$ by subt.

or $x = 225$ or $y = 105$

Whence $z = 130$.

Another, by Master T. Wilson.

First $A + C = B + 250$, $A + B = C + 200$, and $A = C + B - 10$. Then from the two first, $A - B + C = 250$, and $A + B - C = 200$, these added together, give $2A = 450$, or $A = 225$, and the second and third become $B = C - 25$, and $B = 235 - C$, whence by add. $2B = 210$, or $B = 105$ and $C = 130$.

(4) *By* Master G. D. Holcombe.

A farmer sent his servant to the fair with 20£. to buy 20 head of cattle; he bought oxen at 3£. cows at 1£. 5*s*. and calves at 5*s*. per head. How many did he buy of each sort?

Solution by Master A. Washington.

Suppose x the number of oxen, y the cows and z the calves; then, $x + y + z = 20$, and $60x + 25y + 5z = 400s$. Wherefore $z = 20 - y - x = 80 - 5y - 12x$

and $y = 15 - 2x - \frac{3}{4}x$; therefore, in order to have $y =$ a whole number, we must take $x = 4$, then $y = 4$, and $z = 12$.

(5) *By* Master W. BALL, *Plymouth*..

The base of a cone is 3 feet, and its altitude 9; it is required to divide it into two parts, such that the cone cut off by a section parallel to the base, may be to the frustum as 3 : 5.

Solution by Master W. NICHOLSON.

Because the cone cut off is to the remaining part as 3 to 5, it will be to the whole cone as 3 to 8; and, similar solids being to one another as the cubes of their like sides, we have $\sqrt[3]{8} : \sqrt[3]{3} :: 9 : 4.5\ \sqrt[3]{3}$ the altitude of the cone cut off.

Again, by Masters J. ROBSON and ROBERT BELL.

Let $n = .7854$, then the solidity of the whole cone $= 27n$. Put $x =$ the altitude of the cone cut off, then $9 : 3 :: x : \frac{x}{3}$ the diameter of its base; and its solidity $= \frac{nx^3}{27}$; hence the solidity of the frustum $= 27n - \frac{nx^3}{27}$: but by quest. $\frac{nx^3}{27} : 27n - \frac{nx^3}{27} :: 3 : 5$; therefore $\frac{5x^3}{27} = 81 - \frac{3x^3}{27}$, whence $2x = \sqrt[3]{(81 \times 27,)}$ or $x = 4.5\ \sqrt[3]{3}$.

(6) *By* Master W. GODWARD, *Sheffield*.

What two numbers are those, the difference of whose squares is 32, and the sum of their cubes 1072?

Solution by Master C. THOMAS.

Let $x + y$ and $x - y$ represent the numbers; then per question, $4xy = 32$, and $2x^3 + 6xy^2 = 1072$; from the first of these equations, $x = \frac{32}{4y} = \frac{8}{y}$; substitute this

value of x in the second, and we get $\frac{x^2+24}{x^2} - 4xy = \frac{x^2}{y^2}$, or $x^4 - 24x^2 = -24y^2$; hence it is evident that $y = 1$; therefore $x = 5$, consequently, the numbers are 5 and 1.

Question 5, By Master C. Tetley, Enfield.

If a salary of 52*l.* per annum, remaining unpaid for 4 years, amounts to 57*l.* 17*s.* 6*d.* at 3 per cent. per ann. simple interest, how is it payable?

Solution by Master T. Cottell.

Let t be the time, r the rate per cent, and a the annuity; then, by *Wingrave's Arithmetic*, we have $\frac{t^2a - ta}{2} \times r + ta = A$ = the amount; and if n denote the number of payments in one year, we also have (*see note,*) $\frac{n^2t^2a - n^2ta}{2n} \times \frac{r}{n} + \frac{nta}{n} = A$. From this equation n is easily found $= \frac{tar}{t^2ar + 2ta - 2A}$: in the present case, $t = 4$, $a = 13$, $r = .05$, and $A = 57.175$, hence $n = 52$, therefore the salary is payable weekly and $= 13£. \div 52 = 5s.$ per week.

The same by Master J. Greenhough.

Investigation. Let N = any number of years, and n = any number of equal and regular payments, by which any annuity is payable in one year; then Nn will be the number of payments in N years. And, if m = the interest of one pound for one year and a = the annuity, the interest arising from a in N years, will be $\frac{am}{n} + \frac{2am}{n} + \frac{3am}{n}$ to $\frac{(Nn-1)\,am}{n} = \left(\frac{1}{n} + \frac{Nn-1}{n}\right) \times \frac{(Nn-1)\,am}{2n} = \frac{Nam}{2n}(Nn - 1;)$ therefore, $\frac{Nam}{2n}(Nn-1) + Na = A$ = the whole amount due at the end of N years.

From the above eqnation we get $n = Nmu \div (N^2mu + 2Nu - 2A) = 52$, the number of payments in one year.

(8) *By* Master C. WILLIAMS, *Plymouth.*

There is a certain number, to the product of whose digits if you add twice the right-hand digit, the result will be 7 times that digit; and, if to 6 times the sum of the digits, you add 5, it will be the number required. What is the number?

Solution by Master J. WATSON.

Let $x =$ the left hand digit, and $y =$ the right hand one; then $10x + y$ will be the number; also, $xy + 2y = 7y$, and $6x + 6y + 5 = 10x + y$; from the first $x = 5$; therefore, $y = 3$.

Again, by Master J. WHITEHEAD.

Let $10x + y$ be the number, then, $xy + 2y = 7y$, or $x = 5$; also $6x + 6y + 5 = 10x + y$, or, $35 + 6y = 50 + y$, hence $y = 3$, and the number is 53.

(9) *By* Master T. H. MADGE, *Plymouth.*

Required the altitude of a cylinder, when the area of the diagonal section is equal to the square of the diameter of its base.

Solution by Master W. GODWARD.

Put the altitude of the cylinder $= x$, the diameter of its base $= a$; then (Eucl. I. 47.) $\sqrt{(a^2 + x^2)} =$ the transverse diameter of the section, and (if $p = .7854$,) we have $ap\sqrt{(a^2 + x^2)} = a^2$, whence $x = \frac{a}{p\sqrt{(1 - p^2)}}$.

(10) *By* Master R. HOLDSWORTH, *Mr. Nesbit's Academy.*

How many bricks, each 9 inches long, $4\frac{1}{2}$ inches broad, and $3\frac{1}{2}$ inches thick, will build a wall 300 feet long, 14 feet high, and $1\frac{1}{2}$ thick?

Solution by Masters BURLEY and STEAD.

The feet being reduced into inches, we have $\frac{3600 \times 168 \times 13.5}{4.5 \times 9 \times 3.5} = \frac{400 \times 168 \times 6}{7} = 400 \times 24 \times 6 = 57600$ the required number of bricks.

The same by Master T. H. MADGE.

First, $\frac{300 \times 12 \times 14 \times 12 \times 18}{4.5 \times 9 \times 35} = 400 \times 24 \times 8 =$ 76800 the bricks required.

N. B. In the first of these solutions, the wall is supposed to be a brick and a half thick; and in the second, one foot and a half.

Masters J. Greenhough, *Bolton, near Bradford;* J. Watson, *Mr. Swale's Academy, Brunswick-place, Liverpool;* W. Shepherd and C. Thomas, *Eccleshill;* J. Whitehead, *Mr. Garside's Academy, Mirfield;* J. Vint, *Idle;* — Nicholson, *Armley;* T. Cuttell, *Leeds;* A. Washington, and A. Levy, *Mr. Reynard's Mathematical School, Reading;* and J. Nicholson, *Mr. Jaques's School, Penrith,* answered all the questions.

T. H. Madge, *Mr. Treeby's Academy, Plymouth;* B. C. Goodison, and H. Allen, *Rev. H. Davis' Seminary, Eaton Bishop, near Hereford;* and W. Godward, *Free Writing-School, Sheffield,* answered 9 questions.

W Stead and B. Burley, *Wortley Grammar School;* J. Robson and R. Bell, *Mr. Darby's Academy, Sutton;* and T. Wilson, *Free Writing School, Sheffield,* answered 7 questions.

R. Lamb and J. Dodd, *Reading,* answered 5 questions.

A. R. C. Norcott and T. G. Monday, *Reading,* and W. Brown, *Penrith,* answered 4 questions.

Also W. Marsden, R. S. Cotton, G. D. Holcombe, W. Ball, and C. Williams, answered their respective questions.

ANSWERS

TO THE

MATHEMATICAL QUESTIONS

Proposed in the Fourth Number of the first Volume.

(86) *By* Mr. A. NESBIT.

What is the area of the greatest trapezoid that can be inscribed in a semicircle whose radius is 10 chains?

Solution by Mr. JOSHUA GARSIDE, *Mirfield.*

Put $a = 10$, and let the width of the trapezoid be denoted by x; then its area is $x\sqrt{(a^2 - x^2)} + ax = a$ max: the fluxion of which, being put $= o$, and reduced, gives $x = a\sqrt{3} \div 2$; whence $2\sqrt{(a^2 - x^2)} = a =$ the shorter of the parallel sides, and the area $= 3a^2\sqrt{3} \div 4 = 12.990381$ acres.

Again, by Mr. W. WATSON, *Beverley.*

Let $a = 10$, and $2x =$ the sum of the parallel sides; then, the shorter side $= 2x - 2a$, and the perpendicular distance between them is $\sqrt{(2ax - x^2)}$; whence the area $= x\sqrt{(2ax - x^2)}$, which is to be a maximum. By making its fluxion $= o$, we get $2x = 3a$; therefore the area $= 3a^2\sqrt{3} \div 4 = 12.99$ acres.

Otherwise by Mr. J. DARBY, *Sutton.*

It is evident that the longest side of the trapezoid will be equal to twice the radius, or $= 20$; and (Theorem 9th, Simpson *de max. et min.*) the other sides will be equal, and each equal to the chord of 60°, or radius : whence the area $= 300\sqrt{3} \div 4 = 12.99$ acres.

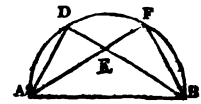
It was also answered by Messrs. R. T. Ambler, J. Baines, J. Crowther, B. Fox, S. Johnson, Sir Danl. de Grandew, I. R. G., Job Shainen, and T. Wilson.

(87) *By* Mr. TREEBY, *Plymouth.*

AB is the diameter of a given circle, BD a given chord; it is required to draw another chord AF intersecting the former in E, so that the triangle BFE may be a *maximum.*

Solution by Sir DANL. DE GRANDEW.

In the annexed figure put BD $= c$, AB $= d$, AD $= \sqrt{(d^2 - c^2)} = b$, and DE $= x$; then will BE $= c - x$, AE $= \sqrt{(b^2 + x^2)}$, and EF $= (cx - x^2) \div \sqrt{(b^2 + x^2)}$.



Also, by similar triangles, AE : AD : : BE : BF $= b\,(c - x) \div \sqrt{(b^2 + x^2)}$: but BF . BE $= b\,(c^2x - 2cx^2 + x^3) \div (b^2 + x^2) = a\ max.$ By making its fluxion $= o$, we get $x^3 + cx^2 + 3\,b^2x = b^2c$, or, by restoring the value of b, $x^3 + cx^2 + 3\,(d^2 - c^2)\,x = cd^2 - c^3$, from which equation x may be found.

The same by Mr. B. FOX, *Wakefield.*

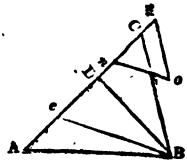
Let d = AB, c = BD, and x = BE; then, DE $= c - x$, AD$^2 = d^2 - c^2$, and AE$^2 = d^2 - 2cx + x^2$. Now, the $\angle$ D and side AD being given, the area of the $\triangle$ADE is as DE or $c - x$, and (Eucl. VI. 19.) that of the $\triangle$BFE as $x^2\,(c - x) \div (d^2 - 2cx + x^2)$, which must, therefore, be a max: from its fluxion put $= o$, we obtain $x^3 - 4cx^2 + (2c^2 + 3d^2)\,x = 2cd^2$, from which, we may find the value of x.

Neat solutions were sent by Messrs. Baines, Crowther, Garside, I. R. G., Johnson, Watson, and the Proposer.

(88) *By* Mr. T. WAINMAN, *Burley-Mill.*

Given the base and an angle at the base, to construct the triangle, when the sum of the squares of the sides *plus* or *minus* their rectangle is equal to the square of the base.

Solution by Mr. S. JOHNSON, *Apperley-Bridge.*



Let ABC be the triangle, and draw BD perpendicular to AC; then will $AB^2 = AC^2 + BC^2 \pm AC \cdot BC$: but (Simpson's Geom. II. 10,) $AB^2 = AC^2 + BC^2 \pm 2CD \cdot AC$, according as BD falls above or below C; therefore, $AC^2 + BC^2 \pm AC \cdot BC = AC^2 + BC^2 \pm 2CD \cdot AC$, or $BC = 2CD$; and because AB and the $\angle$ BAC are given, the construction is easy.

The same by I. R. G.

CONSTRUCTION. Draw the indefinite right line Am making the given $\angle$ with the base AB; in Am take any distance *mn*, upon which describe the equilateral triangle *mno*. Also, draw Bc $\parallel$ *no*, and BC $\parallel$ *mo* meeting Am in *c*, C: then will ABc be the $\triangle$ in the first case, and ABC that in the second.

DEMONSTRATION. Draw BD perpendicular to AC; then, $AB^2 = Ac^2 + Bc^2 + 2Dc \cdot Ac$ (Eucl. II. 12.) $= AC^2 + BC^2 - 2DC \cdot AC$ (ib. 13.) But, by Const. and parallel lines, the triangle *c* BC is equilateral, and $Dc = CD$, or $2Dc = Bc$; and $2DC = BC$; therefore $AB^2 = Ac^2 + Bc^2 + Bc \cdot Ac = AC^2 + BC^2 - BC \cdot AC$. Q. E. D.

COR. The $\angle AcB = 120^0$, and $ACB = 60^0$. — Hence, it appears that the given $\angle$ A must be less than 60^0 in the first case, and 120^0 in the second.

By W. DICKINSON, *High-Town.*

Let ABC be the triangle, AB the given base, and A the given angle. Now, by Quest. $AB^2 = AC^2 +$

$AB^2 = AC^2 + BC^2$ and by Trigon. $AB = AC - BC = \frac{1}{2}AC + \frac{1}{2}AC \times \text{Cos.} \angle C$, wherefore $\frac{1}{2}AC = \frac{1}{2}AC \times \text{Cos.} \angle C$ or $\text{Cos.} \angle C = \frac{1}{2}$: therefore the angle C is 120° in the first case, and 0° in the second, and we have all the angles and a side to determine the triangle. — A well-known Problem.

Two Solutions were sent by Messrs. Barnes, Crowther, Du Louis, of Gainsboro', Fisk, Mannion, and Wilson.

(89) By Mr. W. Wiseman, Hull.

Required the fluent of $\frac{x^5 x}{2 - x}$.

Solution by Mr. J. Baines, Reading.

By Division, the given fluxion $= -x^5 \dot{x} - 2x^4 \dot{x} - 4x^3 \dot{x} - 8x^2 \dot{x} - 16\dot{x} + \frac{32\dot{x}}{2-x}$; the fluent of which is $-\frac{x^6}{6} - \frac{x^5}{5} - \frac{4x^4}{4} - \frac{8x^3}{3} - 8x^2 - 16x - 32 \times \text{h. l.} (2 - x) + C$. Now, if the fluent should vanish when $x = 0$, we have $C = \text{h. l.} 2$, and the required Fluent $= -\frac{x^6}{6} - \frac{x^5}{5} - \frac{4x^4}{4} - \frac{8x^3}{3} - 8x^2 - 16x + 32 \times \text{h. l.} \frac{2}{2-x}$.

By Mr. W. Watson, Beverley.

Put $\frac{x^n \dot{x}}{a - x}$ for the given fluxion; then, (Hutton's Course V. 3, Page 231.) the Fluent is $-\frac{x^n}{n} - \frac{ax^{n-1}}{n-1} - \frac{a^2 x^{n-2}}{n-2} - \frac{a^3 x^{n-3}}{n-3}$ &c, $- a^n \times \text{h. l.} (a - x)$. In the present case $n = 5$, $a = 2$, and the fluent becomes $-\frac{x^5}{5} - \frac{x^4}{2} - \frac{4x^3}{3} - 4x^2 - 16x - 32 \times \text{h. l.} (2 - x)$.

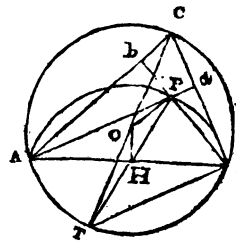
[F] *Nearly in the same manner was the Solution given by Messrs. Charlton, Crowther, Garside, Johnson, and the proposer.*

(90) *By* Mr. J. Baines, *Mathematical School, Reading.*

Given the base, the vertical angle, and the distance from the centre of the circumscribing circle to the intersection of the perpendiculars from the angles upon the opposite sides; to construct the plane triangle.

Solution by Mr. S. Johnson, *Apperley-Bridge.*

Let ABC be the triangle, P the intersection of the perpendiculars A*a*, B*b*, upon the sides BC, AC; O the centre of the circumscribing circle, and CT its diameter; draw AT, BT, PT, and upon AB let fall the perpendicular HO. Now, the base and vertical angle being given, the circumscribing circle is given; wherefore, OT and HO are given lines: and, because AT and BT are respectively perpendicular to AC and BC, APBT is a parallelogram, whose diagonals bisect each other in H; hence (Simpson's Geom. II. 11.) $2HT^2 = OT^2 + OP^2 - 2HO^2$, or HT = a given line; therefore, with centre H, and radius HT, describe an arc cutting the circumference of the circumscribing circle in T; draw the diameter TOC; also, AC, BC; then will ABC be the triangle required. The Demonstration is evident.



By Mr. J. Crowther, *Woodhouse-Grove.*

Analysis. Let ABC be the triangle with its circumscribing circle; P the intersection of the lines A*a*, B*b*, drawn from the extremities of the base, and making *given* angles with the sides. — Because the base AB

and vertical angle ACB are given, the circumscribing circle is known, and its centre O is a given point; also, the $\angle$s P*b*C, *b*C*a*, and C*a*P, being given, the $\angle$ *b*P*a* $=$ $\angle$ APB is given, and the locus of P is the circumference of a given circle passing through A, B; also, because the distance OP is given, the position of the point P becomes known, and the construction evident.

If P be the intersection of perpendiculars from the angles upon the opposite sides, the $\angle$ APB $=$ supplement of the angle ACB.

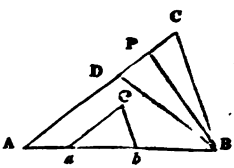
If the circumferences, which are the loci of P, neither touch, nor intersect each other, the Problem is impossible; except when they coincide, and then it is unlimited.

Elegant Solutions were sent by Messrs. Baines, Charlton, Dickinson, Fox, I. R. G. and Watson.

(91) *By* Mr. J. Evans, *Edge-Lane, near Royton.*

It is said in the 16th Theorem, 4th Book of T. Simpson's Geometry, that, "If two triangles (ABC, *abc*) have one angle (A) in the one, equal to one angle (*a*) in the other, and the sides (AB, *ab*, CB, *cb*) about either of the other angles proportional; then will the triangles be equi-angular, provided these last angles (B, *b*) be, either both less, or both greater, than right angles;" which, I say, is erroneous: it is required to shew where the error lies.

Solution by I. R. G.



Let the triangle ABC be similar to the triangle *abc*, and take BD $=$ BC; then, the two triangles ABD, *abc*, have one angle A in the one, equal to one angle *a*, in the other, and the sides AB, *ab*, BD, *bc*, about the angles B, *b*, proportional. But the triangles ABD and *abc* are not equiangular; therefore, if there be a case in which the $\angle$s ABD and ABC are, either both greater, or both

less than right angles, the Theorem is erroneous; if not, it is true. Draw BP perpendicular to AC: Then, it is evident, that the $\angle$s ABD and ABC cannot be both obtuse; therefore, if the $\angle$ ABC be obtuse, the Theorem is true. Again, if the $\angle$ ABC be acute, $\angle$ ABP + $\angle$ CBP is less than a right angle; but, $\angle$ ABP + $\angle$ A = a right angle; therefore, the $\angle$ CBP = $\angle$ DBP must be less than the $\angle$ A, and, in this case, the Theorem is not true, provided the side BC be greater than BP, and less than AB: — but, if BC be equal to BP, or greater than AB, it is true, though the $\angle$ ABC be less than a right angle.

The error lies in having substituted "these last angles B, *b*," for the "remaining angles C, *c*."

Mr. Crowther, after having proved the Theorem to be erroneous, adds;—"The origin of the error will be easily perceived, by referring to II. 17, on which the demonstration of the Theorem in question depends, and in which the sides AB, BC are used instead of AB, AC in the former; though the $\angle$ B was inadvertently left unchanged."

The error was likewise pointed out by Messrs. Baines, Evans the proposer, Johnson, Sir Dan. de Grandew, and Watson: it is also noticed in Emerson's Geom. B. II. 16. schol.

(92) *By* Mr. A. Jaques, *Penrith.*

A cone of dry ash is suspended by its vertex, having 3 feet of its axis immersed in water, which just causes its suspending weight to vanish; but if the cone be suspended by the centre of its base from the same point, the suspending weight will be 4800 ounces. Required the dimensions of the cone.

Solution by Mr. Joshua Garside, *Mirfield.*

In the first place it may be proper to observe, that all cones of the same specific gravity, and of the same height, will sink to the same depth, when put into any given

fluid, as in the question. Now, the required height of a cone of dry ash, that will sink just 36 inches in common water, is, by analogy, found to be 86.706 Inches. Put x = the diameter of this cone's base; then its solidity will be $22.6996x^3$, and being suspended by the centre of its base, that of the part in the water is found $= 1.6248x^2$; also, the specific gravity of dry ash is to that of common water as 4 : 5 :: $1.6248x^2 : 2.031x^2$ the solidity of a cone of wood balancing the weight of the displaced water; therefore, $22.6996x^3 - 2.031x^2 =$ 10368 the solid inches in 4800 ounces of dry ash; whence $x = 22.36$ Inches the diameter of the base.

The same by MR. J. CROWTHER.

Put x = the whole altitude of the cone; then, the specific gravity of the ash, being to that of water as 4: 5; in the first position, the whole cone will be to that above the water as 5 : 1, or $x^3 : (x-3)^3 :: 5 : 1$; therefore, $x = 3\sqrt[3]{5} \div (\sqrt[3]{5} - 1) = 7.2255$ the altitude. Again, in the second position, putting w = weight of the cone, we have $3^3 : w - 4800 :: \frac{4}{5}(7.2255)^3 : w$; hence $w = 5271.64$ oz. $= 6.5895$ cubic feet of ash; whence the diameter of the base is found $= 1.866$ft.

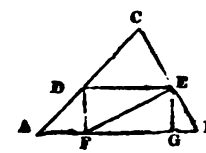
It was also answered by Messrs. Baines, I. R. G. Sir Dan. de Grandew, Johnson, Job Shainen, Watson, and the Proposer.

(93) *By* GEOMETRICUS, *Rotherham.*

In a given triangle to inscribe a rectangle, whose perimeter shall have a given ratio to its diagonal.

Solution by MR. S. JOHNSON.

ANALYSIS. Because the ratio of the perimeter to the diagonal is given, the ratio of its half DE + DF to EF is given, and $\angle$ EDF being given, the triangle EDF is given in species (Dat. 48.) and the ratio of DE



to DF is known: but the ratio AD : DF is given, hence, that of AD to DE is known (Dat. 9.) also, the ratio DC : DE being given, that of AD : DC is given, (ib.) But, AC is given, therefore the point D is given, and the construction obvious.

From the above analysis it is evident, that if DEFG be *any* parallelogram whose angles are given, the point D will be given, and the construction effected by drawing DE || AB, DF making the given ∠ with DE, and EG parallel to DF.

The same, by MR. J. CROWTHER.

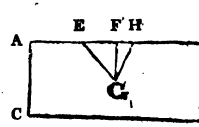
Let ABC be the given triangle, EF the diagonal of the inscribed rectangle; then, the ratio of EF to 2 (DE + DF), and consequently, of EF to DE + DF, is given. Hence the triangle DEF is given in species, the ratio of DE to DF given, and the rectangle may be inscribed by Prob. 1. Simpson's Geom.

Neat Solutions were sent by Messrs. Dickinson, Fox, I. R. G., Watson, and Wright.

(94) *By* MECHANICUS, *Leeds.*

A cylinder, suspended by a string fixed to its side, made an angle of 45° with the string; but having removed the point of suspension 1½ inch, in the direction of its axis, the angle was 63° 26′ 6″, and the opposite end up. Determine the point of suspension when the cylinder is parallel to the horizon.

Solution by SIR DAN. DE GRANDEW.



Let ABCD represent the cylinder, G its centre of gravity, E the point of suspension in the first position, H that in the second, and F the required point. Then, will GF be perpendicular to AB, the ∠ GEF = 45°, the ∠ GHF = 63° 26′ 6″ and HE = 1.5. Wherefore, in

the triangle GEH, are given all the angles and side HB, to find the segment EF = 1 Inch = the distance of the required point, from the first point of suspension, or, FH = .5 = its distance from the second.

Another by MR. W. WATSON, *Beverley.*

Let ABCD represent a section of the cylinder, G its centre of gravity, and E, H, the points of suspension, causing the cylinder to make, with the string, the ∠ GHF = 63° 26′ 6″, and the ∠ GEF = 45°. Draw GF perpendicular to AB; then will F be the required point, and we have HF × tan. GHF = GF; also EF × tan. GEF = GF. Wherefore, HE × tan. GHF = EF × tan GEF; but EF = HE — HF; therefore HF × tan. GHF = (HE — HF) × tan. GEF; whence, HF = HB × tan. GEF ÷ (tan. GHF + tan. GEF) = ½, or EF = 1 Inch.

Again by MR. J. BAINES, *Reading.*

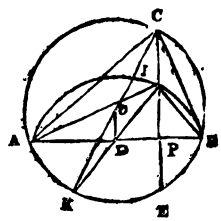
Let $a = 1.5$, $m =$ co-tan. of 45°, $n =$ Co-tan. of 63° 26′ 6″, and $x =$ the radius of the cylinder; then, $1 : m :: x : mx$ the distance of the first point of suspension from the middle of the cylinder; and, $1 : x :: n : nx$ the distance of the second. Therefore $mx + nx = a$, or $x = \frac{a}{m+n}$, and $mx = \frac{am}{m+n} = 1$ Inch, the distance at which the string must be fixed from the first point.

It was likewise answered by Messrs. Crowther, Fox, Garside, I. R. G., Johnson, Job Shainen, and Watson.

(95) *By* Mr. J. CROWTHER, *Woodhouse-Grove.*

Given the vertical angle, the distance from the base to the intersection of the perpendiculars upon the opposite sides, and the ratio of the rectangle of the sides to the rectangle of the segments of the base made by a perp. from the vertical angle; to construct the plane triangle.

Solution by MR. CROWTHER, *the Proposer.*



Let ABC be the triangle with its circumscribing circle, I the intersection of the perpendiculars; produce CP to meet the circle in E; then, the ∠ AIB being the supplement of the ∠ ACB, the locus of I is the circumference of a circle, the segment of which AIB is similar and equal to the segment AEB, and, because IP is perpendicular to AB, it is equal to PE. Again, by Quest. AC . CB : (AP . PB) PE. CP :: m : n, or Diam. × CP : (PE) PI . CP :: m : n :: Diam. : PI; hence the Diameter of the circle ACBE is known; and, the ∠ ACB being given, AB is a given line; but PE is given; therefore, E is a given point, and the Construction manifest.

The same by MR. F. CHARLTON, *Kirkheaton, Northumberland.*

ANALYSIS. Suppose it done, ABC the triangle, I the intersection of the perpendiculars, and CK the diameter of the circumscribing circle whose centre is O. Then, the ratio of AB . BC to AP . PB is given; but AC . BC = CK. CP, and AP. PB = CP. PI; therefore, CK. CP : CP . PI, or CK : PI is a given ratio; but, as PI is given, CK is known, and consequently, the base AB and perpendicular OD are given lines; but CI = 2OD; therefore CP is given, and the construction obvious.

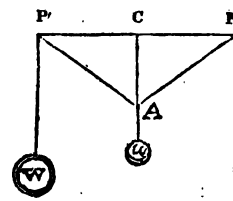
It was also answered by I. R. G., and Mr. Johnson.

(96) *By* Mr. G. DUCKET, *Northwich.*

If a string 20 feet long be made fast at one end, the other end passing over a pulley at the distance of 10 feet in a horizontal position, to which is connected a weight of 4lb. Also a weight of 2lb. is placed between

the pulley and fixed end, so as to slide freely. Required the greatest velocity of the greater weight in its descent, when its motion commences at the pulley; and their position when at rest. The weight of the string being inconsiderable.

Solution by AMICUS.



Let P be the point to which the string is fastened, P′ the pulley, and W, w, the weights. Draw AC perpendicular to P′P, then will P′C = CP: put CP = a, AC = x, AP′ = y, and s = the velocity which gravity can generate in a determinate unit of time, as a second. Also, let $\dot{t}$ denote the moment of time in which W descends through the space $2\dot{y}$; then the velocity of W is $\frac{2\dot{y}}{\dot{t}}$, and (supposing $\dot{t}$ constant) the variation of that velocity $= \frac{2\ddot{y}}{\dot{t}^2}$; therefore the motive force of W is $W \times \frac{2\ddot{y}}{\dot{t}^2}$; but, if it were not connected to the string, its motive force would be $W \times s$; consequently the quantity of motion lost, by being connected with w, is $W \times \left(s - \frac{2\ddot{y}}{\dot{t}^2}\right)$ = the tension of the string. Again, the velocity with which w ascends is $\frac{\dot{x}}{\dot{t}}$, and the variation of that velocity is $\frac{\ddot{x}}{\dot{t}^2}$ = the accelerating force in the direction AC; but, if w were at liberty, its accelerating force (in the same direction) would be $-s$;

therefore the motive force, which causes w to ascend with the velocity $\frac{\dot{x}}{\dot{t}}$, is $w \times \left(s + \frac{\ddot{x}}{\dot{t}^2}\right)$, acting in the direction AC, and is produced by the tension of the string; therefore $2x : y :: w \times \frac{y}{2x}\left(s + \frac{\ddot{x}}{\dot{t}^2}\right) =$ that tension.

Hence, $W \times \left(s - \frac{2\ddot{y}}{\dot{t}^2}\right) = w \times \frac{y}{2x}\left(s + \frac{\ddot{x}}{\dot{t}^2}\right)$,

where $y^2 = a^2 + x^2$. Now, $y\dot{y} = x\dot{x}$, or $\frac{y}{x} = \frac{\dot{x}}{\dot{y}}$, and, if $m = W \div w$, we have, by substitution, &c.

$$s\,(2my - x) = \frac{4m\ddot{y}y + \dot{x}\ddot{x}}{\dot{t}^2}$$

and by taking the fluents ($\dot{t}$ being constant) we obtain $s\,(2my - x) + c = \frac{4my^2 + x^2}{2\dot{t}^2}$, c being the necessary correction. Again, by substituting $\frac{y^2\dot{y}^2}{x^2}$ for $\dot{x}^2$, and v for $\frac{2\dot{y}}{\dot{t}}$ the velocity corresponding to y, we get

$$v = \frac{2\dot{y}}{\dot{t}} = 2x \times \left(\frac{4smy - 2sx + 2c}{4my^2 - 4ma^2 + y^2}\right)^{\frac{1}{2}}$$

Let $2l$ = the length of the string, then at the commencement of the motion, we have $y = l$, $x = \sqrt{(l^2 - a^2)} = p$, and $v = 0$; therefore $4sml - 2sp + 2c = 0$, and $2c = 2sp - 4sml$; wherefore the general expression for the velocity of W is

$$v = 2\sqrt{(y^2 - a^2)} \times \left(\frac{2sp - 2s\sqrt{(y^2 - a^2)} - 4sm(l - y)}{4my^2 - 4ma^2 + y^2}\right)^{\frac{1}{2}}$$

Again, if we suppose the weights to be at rest; 2AC : AP :: w : W, or, $w^2y^2 = 4W^2y^2 - 4W^2a^2$, and $y^2 = \frac{4a^2m^2}{4m^2 - 1}$; hence x, or the distance of the weight w from the horizontal line P'P, will be $a \div \sqrt{(4m^2 - 1)}$. By making the fluxion of the velocity $= 0$, we find the value of y which gives the greatest velocity of W.

The same by URANIO, H. U. GOTT.

Let P be the pulley, which we will suppose reduced to its axis and void of friction, P the point to which the end of the string is attached, and W, w the weights. Also, let v be the velocity with which W descends, and u that of w upwards, y = AP', x = AC, a = P'C = CP, t = the tension of the string, and s = the accelerating force of gravity.

The motive force which would be generated in W, if not connected to the string, would be $W \times s$; but the tension of the string being t, the real momentum of W is $W \times s - t$, which must be equal to $W \times \frac{\dot{v}}{\frac{2y}{v}} = W \times \frac{v\dot{v}}{2y}$; therefore, $t = W \times s - W \times \frac{v\dot{v}}{2y}$: Now, t is the motive force, which causes w to ascend, and $y : 2x :: t : \frac{2tx}{y}$ = the force which opposes the descent of w, therefore

$$\frac{2x}{y}\left(W \times s - W \times \frac{v\dot{v}}{2y}\right) - w \times s = w \times \frac{u\dot{u}}{\dot{x}};$$

but $x^2 + a^2 = y^2$, *or* $x\dot{x} = y\dot{y}$; hence $\frac{x}{y} = \frac{\dot{y}}{\dot{x}}$ and we have $\frac{2\dot{y}}{\dot{x}}\left(W \times s - W \times \frac{v\dot{v}}{2y}\right) = w \times s + w \times \frac{u\dot{u}}{\dot{x}}$

or, $2W \times s\dot{y} - w \times s\dot{x} = W \times v\dot{v} + w \times u\dot{u}$; and by taking the fluents, &c. we have $2c + 4W \times sy - 2w$

$\times sx = W \times v^2 + w \times u^2$: but $\dot{vx} = 2\dot{uy}$, or, $\dot{u} = \frac{\dot{vx}}{2y} = \frac{v\dot{y}}{2x}$

this, being substituted for u in the above equation, we get $v = 2x\left(\frac{4W \times sy - 2w \times sx + 2(}{4W \times x^2 + w \times y^2}\right)^{\frac{1}{2}}$, the same as before.

It was also answered by Mr. J. Crowther.

(102) *By* Mr. HIRST, *Marsden.*

Given $5x + 7y + 11z = 224$, to find all the possible values of x, y, and z, in whole positive numbers.

Solution by Mr. CROWTHER.

Here $x = \frac{224 - 7y - 11z}{5} = 45 - y - 2z - \frac{1 + 2y + z}{5} = wh$: therefore, $\frac{4y + 2z + 2}{5} = wh$;

$\therefore y = 5p - 3z + 2$, and $x = 42 + 2z - 7p$.

If $p = 1$,	2,	3,	4,	5,	6, &c.	1, 2, 3, 4, 5, 6, &c.
$z = 1$	$x = 37$,	30,	23,	16,	9, 2 &	$y = 4$, 9, 14, 19, 24, 29
2	39,	32,	25,	18,	11, 4,	1, 6, 11, 16, 21, 26
3	34,	27,	20,	13,	6,	3, 8, 13, 18, 23
4	29,	22,	15,	8,	1	5, 10, 15, 20
&c.			&c.			25, &c.

The whole number of answers will be 59; not 60, as is erroneously stated in Simpson's and Bonnycastle's Algebra. The error is where $z = 14$, in which case 7 and 5 are the only positive values of x and y respectively.

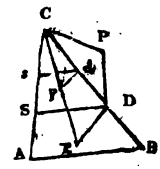
It was also answered by Messrs. Baines, Blaymires, Garside, and Johnson.

(98) *By* Mr. W. SETTLE, *Holbeck.*

ABC is a given triangle, and P a given point; it is required to determine, *geometrically*, a point D in BC, so, that joining PD and drawing DS parallel to AB terminating in the side AC, the ratio of PD to DS may be given.

Solution by Mr. S. Johnson.

Suppose D the point which was to be found; Then, SD being parallel to AB, the ratio of DS to DC is given; hence, that of DP to DC becomes known; and, because DCP is a given angle, the triangle DCP is given in species: (Dat. 47.) but CP is given, whence CD, and consequently the position of the point D, may be easily determined.



The same by Mr. J. Crowther.

Construction. Join CP, and draw $ds \parallel AB$ meeting AC in s, and BC in d; from d to CP, apply dp so that $n : m :: ds : dp$ ($m : n$ being the given ratio.) Then, PD being drawn parallel to pd, will meet BC in D the required point.—For, draw DS $\parallel ds$ meeting AC in S; then, by Const. and simi. tri. $n : m :: ds : dp :: DS : DP$; therefore; $m : n :: DP : DS$ the given ratio; also, DS, being parallel to ds is parallel to AB.

It was also ingeniously answered by Messrs. Baines, I. R. G. Settle, and Watson.

(99) *By* Mr. R. Maffett, *Plymouth.*

Given the hypothenuse of a right angled triangle, to construct it *geometrically*, when the solid, under the square of the perpendicular from the right angle upon the hypothenuse, and one of the segments thereof, is a *maximum.*

Solution by Mr. Watson, *Beverley.*

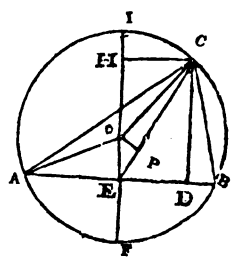
In any right angled triangle, the square of the perpendicular from the right angle upon the hypothenuse, is equal to the rectangle of the segments made thereby; wherefore, one of the segments multiplied by the square of the other, must be a maximum; therefore, (Simpson *de max. et min.* 17.) the former must be half of the latter—whence the construction is evident.

Almost exactly in the same manner was the solution given by Messrs. Baines, Charleton, Crowther, Johnson, Job Shainen, and the proposer.

(100) *By* Mr. BUTTERWORTH, *Haggate, near Oldham.*

Given the base and perpendicular to construct the triangle, when the line bisecting the base has to the perpendicular demitted from the centre of the circumscribing circle upon the base, the given ratio of $m : n$.

Solution by MR. W. WRIGHT, *Bonby, near Barton-upon-Humber.*



ANALYSIS. Suppose ACB the triangle sought, O the centre of its circumscribing circle, IF its diameter bisecting AB in E; join CE, and draw CH perpendicular to EI. Then, $AE^2 = AO^2 - OE^2 = CO^2 - OE^2 = CH^2 + HO^2 - OE^2 = CH^2 + (HE - OE)^2 - OE^2 = CH^2 + HE^2 - HE.OE = CE^2 - 2HE.OE$; but by Quest. $CE : OE :: m : n : s :$ 1 a given ratio, or $CE = s.OE$; hence $s^2.OE^2 - 2HE.OE = AE^2$, and, as HE and AE are both given by the Ques. the method of determining OE is well known, and the construction easy.

The same by MR. J. CROWTHER.

ANALYSIS. Let ABC be the triangle, CD the perpendicular from the vertical $\angle$ upon the base, OE that from the centre of the circumscribing circle: join AO, CO, CE, and draw OP perpendicular to CE; then, because OE is parallel to CD, the right angled triangles, OEP and ECD are similar. But by Quest.

$m : n :: CE : OE :: CD : EP = \frac{n}{m} \cdot CD =$ a given line.

Again, $CP^2 + OP^2 (OE^2 - EP^2) = OC^2$, and $AE^2 + OE^2 = AO^2 = OC^2$; wherefore, $CP^2 + OE^2 - EP^2 = AE^2 + OE^2$, or, $CP^2 = AE^2 + EP^2$ a given quantity: hence CP, and consequently $CE = CP + EP$, is known, and the construction manifest.

Again by Mr. S. Johnson.

Analysis. Suppose ACB the required triangle, O the centre of the circumscribing circle whose diameter IF is perpendicular to AB the given base; OE, CD the perpendiculars upon AB: join C, E, and draw CH perpendicular to IE, and OP perpendicular to CE; then, by similar triangles, EH (CD): EP :: CE : OE :: m : n a given ratio, and, EH being given, EP is known. Again, $CP^2 - PE^2 = OC^2 - OE^2 = AO^2 - OE^2 = AE^2$, or, $CP^2 = AE^2 + PE^2$ = a given quantity, and $CE = PE + CP$ = a given line; hence the

Construction. Bisect the given base AB in E, erect the perpendicular EH = the given one, draw HC ∥ AB; with radius = PE + CP and centre E, describe an arc cutting HC in C; then, AC, BC, being drawn, ABC will be the triangle required. The Demonstration is evident from the Analysis.

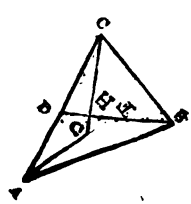
Elegant Solutions were sent by Messrs. Butterworth, the proposer, Charlton, and I. R. G.

(16) *By* Mr. W. Putsey, *Pickering.*

Suppose a conical glass of no sensible weight, whose diameter is 3 inches and depth 6, to be freely suspended by its edge: how much water will it hold in this position?

Solution by Mr. W. Settle, *Holbeck.*

Let ABC represent a section of the glass passing through its axis, and DB the surface of the water; bisect DB in F, and join A, F; also, from C the point of suspension, let fall the perpendicular CH, and produce it to meet AF



in G, which (because the weight of the vessel is not considered) will be the centre of gravity of the contained fluid, when at rest. Put AB = AC = a, BC = d, CD = x, and AD = $a - x$; then, (Lemma, page 128, Simp. Select exercises) $DB^2 = (ad^2 - xd^2 + ax^2) \div a$, which call $4z^2$, and (Simp. Geom. II. 12.) $AF^2 = a^2 - ax + \frac{1}{2}x^2 - z^2$, which put $= v^2$; also, (Simp. Select Exercises Prob. 23.) $DH = (x^2 + 4z^2 - d^2) \div 4z$: hence $HF = DF - DH = (d^2 - x^2) \div 4z$. Again, in the triangle DFA, we have all the sides to find the co-sine of the $\angle F = (z^2 + v^2 - a^2 + 2ax - x^2) \div 2vz$ = sine of the $\angle HGF$; then, sine of the $\angle HGF$: HF : : Radius : $FG = v(d^2 - x^2) \div 2(z^2 + v^2 - a^2 + 2ax - x^2)$; but G being the centre of gravity of the cone ABD, FG is also equal to one fourth of AF, therefore

$$\frac{v(d^2 - x^2)}{2(z^2 + v^2 - a^2 + 2ax - x^2)} = \frac{v}{4}$$

This, by restoring the value of v, &c. becomes $3x^2 + 2ax = 4d^2$: whence $x = \frac{\sqrt{(12d^2 + a^2)} - a}{3}$; in the present case, $d = 3$, $a = \sqrt{38.25}$, and $x = 1.9695$. Hence the content of the ungula or vacuity CBD = 6.2038, and that of the whole glass = 14.1372; their difference = 14. 1372 — 6.2038 = 7.9334 solid inches, is the quantity of water, which the vessel will contain, when suspended by its edge.

(102) *By* ANALYTICUS, *Leeds.*

Find general expressions for x, y, and z, such that $x^2 + y^2$, $z^2 + 3x^2$, and $z^2 + 3y^2$ may be squares.

Solution by MR. S. JOHNSON.

Let it be required to make $x^2 + y^2$, $z^2 + ax^2$, and $z^2 + ay^2$, squares. First, let $z^2 + ax^2 = b^2$, $z^2 + ay^2 = c^2$; then $z^2 = b^2 - ax^2 = c^2 - ay^2$; put $rv - b = c$, $sv - x = y$, and we get $r^2v^2 - 2rvb + b^2 = as^2v^2 + 2asvx - ax^2 = b^2 - x^2$, from which, $v = (2asx - 2rb) \div (as^2 - r^2)$; wherefore, $c = (2arsx - bas^2$

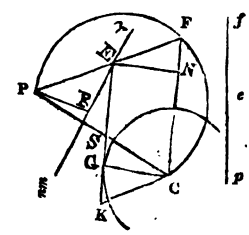
$-br^2) \div (as^2 - r^2)$ and $y = (asx^2 + r^2x - 2rsb) \div (as^2 - r^2)$. Now, $z^2 = b^2 - ax^2$; and in order to avoid fractions, let $b = (n^2 + am^2).(as^2 - r^2)$, $x = 2nm(as^2 - r^2)$; then, y becomes $= 2nm(as^2 + r^2) - 2rs(n^2 + am^2)$, and there remains $(x^2 + y^2) \div 4$ to be made a square; this, by substitution becomes $a^2r^2s^2m^2 - 2anm^3(as^3r + r^3s) + 2n^2m^2(a^2s^4 + ar^2s^2 + r^4) - 2n^3m(ars^3 + r^3s) + r^2s^2n^4$: let its root be assumed $= rsn^2 + nm(as^2 + r^2) - arsm^2$; then, $a^2r^2s^2m^4 - 2anm^3(as^3r + r^3s) + 2n^2m^2(a^2s^4 + ar^2s^2 + r^4) - 2n^3m(ars^3 + r^3s) + r^2s^2n^4 = (rsn^2 + nm(as^2 + r^2) - arsm^2)^2 = a^2r^2s^2m^4 - 2anm^3(ars^3 + r^3s) + n^2m^2(as^2 + r^2)^2 - 2ar^2s^2n^2m^2 + 2n^3m(as^3r + r^3s) + r^2s^2n^4$, this, by reduction, becomes $m(2a^2s^4 + 2ar^2s^2 + 2r^4) - 4n(as^3r + r^3s) = m(a^2s^4 + r^4)$, or $m(as^2 + r^2)^2 = 4n(as^3r + sr^3)$; hence $m(as^2 + r^2) = 4nrs$; therefore, we may take $m = 4rs$ and $n = as^2 + r^2$, where s and r may be taken at pleasure. In the present case, $a = 3$; let $r = 2$ and $s = 1$; then $m = 8$, $n = 7$, therefore, $x = 112$, $y = 180$, $z = 143$.

(103) *By* Mr. J. KAY, *Royton.*

C is the centre of a given circle, CP is given in length, and *nr* is a line given in position, cutting PC in the given point *m*; it is required to draw the tangent GE to touch the circle in G, meet *nr* in E, and cut PC in S so, that PE^2 may be equal to P*m*.PS.

Solution by the PROPOSER.

CONSTRUCTION. From P draw PR perpendicular to *nr*, find a line *ef* that is a fourth proportional to PR, P*m*, CG; to *ef* add a line *pe* so, that *pe* . *pf* may be equal to P*m*. PC; from P apply PE to *nr* and equal to *pe* the part added to *ef*; and from E draw the tangent EG, and the thing is done.



DEMONSTRATION. Produce PE to F, making PF = *pf*; then PF . PE = *pf* . *pe* because PE = *pe* by const. also *pf* . *pe* = P*m* . PC (ib.) therefore, PF . PE = P*m*. PC, or, PF : PC :: P*m* : PE, hence the triangles P*m*E, PFC are similar, and the ∠ P*m*E = ∠ PFC. Draw EN perpendicular to CF, then, by similar triangles, PR : P*m* :: EN : EF; but, by const. PR : P*m* :: GC : *ef* (EF) . therefore, CG = EN, consequently, GE is par. to CF, or, the ∠ GEP = ∠ CFP = ∠ P*m*E; hence the triangles P*m*E, PES are similar, and P*m* : PE :: PE : PS, therefore, PE^2 = P*m* . PS. Q. E. D.

The same by MR. W. WRIGHT, *Bonby, near Barton-upon-Humber.*

ANALYSIS. Suppose the thing done, join C, G, and par. to PE draw CK cutting EG prolonged (if needful) in K, produce PE to F making EF = CK, and draw CF. By the quest. EP^2 = P*m* . PS, or, P*m* : EP :: EP : PS, hence the triangles PF*m*, PSE, are sim. and the ∠ PES = ∠ P*m*E, and as EF, CK, are equal and par. CF is par. to EK, and the triangle PFC is sim. to PES, and thence to P*m*E, hence, PF : CP :: P*m* : EP, or, PF . PE = CP . P*m*, and, as CG is given, and by parallels, the ∠ CKG = ∠ PES = ∠ P*m* E, we have CK = EF given, and CP, P*m*, being known, EP may be found by lineal sections, and the following const. is very obvious.

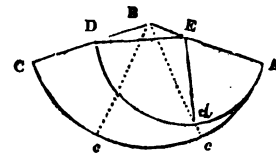
Having determined EP, upon CP describe an arc to contain an angle CFP = ∠ P*m* E, inscribe the chord PF = EP + EF cutting *m*r in E, draw the tangent EG, and it is done.

(104) *By* Mr. BUTTERWORTH.

Suppose one end of a string to be fastened to a given point on the surface of a right cone; and the other extremity of the string brought into a line with the vertex and given point, kept tight, and carried 3 times round the cone, whose surface is supposed to be perfectly polished. It is required to determine the length

of the curve traced out by the extremity of the string upon the surface of the cone; when the slant side of the cone, the diameter, the length of the string, and the distance of the given point from the vertex, are 12, $8\frac{1}{2}$, 10, and 2 respectively.

Solution by MR. W. SETTLE.



Let us suppose three times the surface of the cone to be extended on a plane as in the annexed figure, B the vertex, AE = ED the string fixed to the point E; then, because the string is kept tight and the surface of the cone perfectly polished, the point D will always be at the greatest distance possible from the point E; hence it is evident, that DE will be a right line on the extended surface, and being given, the locus of D will be a given circle.

Let the point A be supposed to have passed over a given arc Ac, and the end of the string to have arrived at d in the radius Bc; then, the arc Ac and AB = Bc being given, the $\angle$ ABc is given, and we have two sides (BE, Ed,) and an opposite angle (B) to find the included angle (E;) whence the $\angle$ AEd is known, and consequently the length of the arc Ad. In the present case the arc AC = 157° 30′ = the $\angle$ DBE, BE = 2. DE = 10, whence the $\angle$ DEB = 18° 6′ 38″, or, the $\angle$ DEA = 161° 53′ 22″, and 180° : 160° 53′ 22″ : : 10 × 3.1416 : 28.27 the length of the curve required.

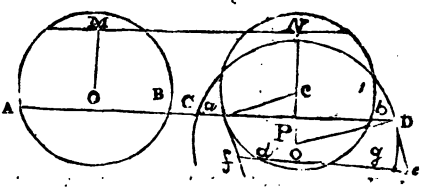
It was also answered by Mr. Butterworth, the proposer.

(105) *By* GEOMETRICUS, *Rotherham.*

To draw geometrically, a straight line AD parallel to a line given in position cutting two given circles in the points A, B, C, D, so that intercepted, chords AB CD, shall have a given ratio, or that their sum AB + CD shall be a *maximum*.

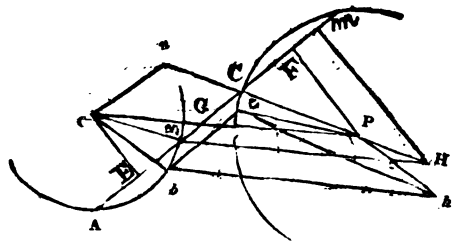
Solution by MR. W. WRIGHT, *Bonby, near Barton-upon-Humber.*

CASE I. ANALYSIS. Suppose it done; from O, *o*, the centres of the given circles draw ON, *o* M, perp. to the line given in position; in ON take *c* N = *o* M; with centre *c* and radius = A*o* or *o*B describe the circle *abd*, cutting AD in *a*, *b*; draw *ac* and CD; Then we shall have *ab* = AB, and CD : *ab* or their halves DP: *a*P: or the squares thereof DP^2 : aP^2 a given ratio, but $OD^2 - OP^2 = DP^2$, and $ac^2 - cP^2 = aP^2$: hence $OD^2 - OP^2$: $ac^2 - cP^2$ a given ratio; and as OD, *ac*, and evidently OP + *c*P = O*c* are given, the problem is easily reduced to a well known case of lineal sections, viz. to determine OP when $OP^2 + m$ (OC. OP) = a given space, which being done, through P draw AD par. to MN, and it is done.



CASE. 2. When tangents *af*, D*e*, drawn to the circles at *a* and D are par. then is *a*D or the double thereof CD + *ab* = CD + AB a maximum; for par. to MN draw *fe* cutting the circles in *g* and *d* respectively; then is *dg* less than *fe* or its equal *a*D, also by par. lines *a*D makes = ∠ s with the tangents, and *ac*, OD, are perp. to them respectively, hence ∠ *ca*P = OPD, and the triangles *ca*P, ODP, are sim. and OP: *c*P : : OD : *ac* a given ratio, hence P is given, and drawing AD, through it, par. to MN, it is done.

The same, by MR. J. CROWTHER.



1. When the chords have a given ratio. ANALYSIS. Let, O, P, be the centres of the 2 circles, AD the line drawn parallel to that given in position; so that AB : CD may be a given ratio, suppose $m : n$. Join OP cutting AD in G : draw OE, PF perpendicular to AD ; also join O,B ; C,P. — Then, $m : n$ :: AB : CD : : BE : CF ; or as $m^2 : n^2$: : BE^2 (BO^2 — OE^2) : CF^2 (PC^2 — PF^2) ; therefore m^2 . (PC^2 — PF^2) = n^2 . (BO^2 — OE^2) . But because $\angle$ OGE = $\angle$ PGF is given, the ratio OE : OG = FP : PG is given . — Let OG : OE : : 1 : r ; then $OE^2 = r^2$. OG^2, and $PF^2 = r^2$. PG^2 ; therefore m^2. PC^2 — (m^2 . PF^2) $m^2 r^2$. PG^2 = n^2. BO^2 — (n^2. OE^2) $n^2 r^2$. OG^2; or $n^2 r^2$. OG^2 ∞ $m^2 r^2$. PG^2 = n^2. BO^2, ∞ m^2. PC^2, a given magnitude : the Construction will then be the same as that given by Geometricus of Rotherham to No. 74.

2. When the sum of the chords AB + CD is a max. Because OE, PF are always parallel to each other, and so perpendicular to AD, EF the perpen. dist. between them is constant. — Hence, AB + CD, or BE + CF will be a maximum, when BC is a minimum. We have then this

CONSTRUCTION. Draw the indefinite right line On parallel to the line given in position ; from P to On apply Pn = the sum of the radii of the two circles, cutting the circle P in C, through C parallel to On, draw AD, cutting the 2 circles in A, B and C, D respectively : then will BC be a minimum.

THE

LEEDS CORRESPONDENT,

A

Literary, Mathematical and Philosophical Miscellany.

Tarda fluunt ingrataque tempora, quæ spem
Consiliumque morantur agendi gnaviter id, quod
Æquè pauperibus prodest, locupletibus æquè;
Æquè neglectum pueris senibusque nocebit.

HORAT.

No. II. JULY, 1816. Vol. II.

Themes and translations are respectfully requested of those among our junior correspondents, who have not attained to sixteen years of age.

JUVENILE DEPARTMENT

THE M .

The laudable Desire of Fame.

TRUE GLORY.

True Glory consists in that praise or approbation, which is received from the world in consequence of the performance of great and virtuous actions, that are beneficial to mankind. It appears to have been the intention of man's Omniscient Creator, by engrafting in him a desire of obtaining the applause of his fellow-creatures, to excite in him a wish of effecting those noble and splendid deeds which are advantageous either to the world in general, the country which he inhabits, or the community within whose limits he resides. And although there be many who are actuated by different

motives, both with regard to their views and their methods of obtaining glory, yet their various endeavours may be all for one common end, and modified and accommodated so as to correspond with their respective ranks and conditions in society. It does not fall to the lot of every man to perform those actions which are of splendid utility in an extensive sphere; yet it is in the power of every man, from the king on his throne to the meanest of his subjects, to form good designs, which, when executed, may be of the greatest benefit in a large or a small circle. Moreover, true glory has the advantage of appearing more pleasing and more worthy of honour, the more closely and strictly it is examined; whereas false glory, however splendid the form that it has assumed, whenever a critical examination takes place, is quickly detected and its weak votary branded with disgrace.

The Greeks and the Romans have produced many noble and exemplary instances of true glory: Amongst which, that of Marcus Attilius Regulus, the Roman Consul, stands in the first rank,—who, being taken prisoner by the Carthaginians, was by them sent to his fellow-citizens, to offer some proposals for the accommodation of existing differences and an exchange of prisoners. Conceiving these overtures to be prejudicial to the interest of his country, Regulus, on his arrival at Rome, persuaded the senate to reject them. He then, according to the terms of his agreement, returned to Carthage, where, in consequence of the unfavourable result of his embassy, he was barbarously and shamefully put to death for thus preferring his country's welfare to his own life.

For the preservation of true glory, after it has been once obtained, Addison observes, that there is only one thing necessary, which is,—to hear the opposers of it with patience, and preserve the virtue by which it is acquired: When a man is thoroughly persuaded, that he ought neither to admire, wish for, nor pursue any thing but what is exactly his duty, it is not in the

power of seasons, persons, or accidents to diminish his value. Triumph, applause, and acclamations are dear to the mind of man; but it is still a more exquisite delight to say to yourself that you have done well, than to hear the whole world pronounce you glorious. Their commendation is vain, except you can yourself join them in your own reflections.

As it appears, then, that the path of glory is open to every rank and condition amongst men, and that every one can more or less perform actions which are of advantage to his fellow-creatures, it is a duty incumbent on all who either wish or profess themselves to be worthy members of society, to endeavour to acquire TRUE GLORY by performing virtuous actions, and to impress it upon others both by precept and example.

Liverpool. JOHN WATSON.

CERTITUDE DE LA MORT.

Le jour de la mort est le juge de tous les autres jours; c'est l'instant de recueillir le fruit de nos études. Une belle mort jette un éclat immortel sur tout le cours de la vie, comme une mort infâme la déshonore à jamais. On demandoit à Epaminondas, lequel il estimoit le plus de Chabrias, d'Hippocrate, ou de lui-même. Il répondit: "avant de résoudre cette question, il faut "nos voir mourir tous trois." La raison est qu'on peut feindre jusqu'à ce dernier moment.

Le meilleur usage qu'on puisse faire de la vie, est d'apprendre à mourir. On craint la mort comme le pire de tous les maux. Pourquoi ne pas l'attendre doucement, patiemment, comme une chose inévitable, naturelle, raisonnable? Comment peut-on savoir si la mort est un mal? Personne ne s'en est plaint. Les prédicateurs nous en font d'horribles peintures; c'est le remords, le crime, qu'ils nous représentent; et non la mort. Encore une fois, pourquoi la craindre? On ne sait ce que c'est. Aussi un sage disoit que

craindre la mort, c'est feindre de savoir ce que tout le monde ignore. On est bien malheureux, quand on trouble sa vie par la crainte de la mort, et sa mort par l'amour de la vie.

Quoi de plus naturel que la mort? Se fâcher de mourir, c'est se fâcher d'être homme. Quoi de plus fou que de se tourmenter pour un mal inévitable? Va-t-on frapper à une porte qui ne s'ouvre jamais? Qu'y a-t-il de plus inéxorable que la mort? On remédie aux maux incertains, et l'on se soumet aux maux irrémédiables. En effet, quelles plaintes, quels murmures insolens n'entendroit-on pas contre la nature, si nous ne devions jamais mourir? Qu'on maudiroit son sort, s'il ne devoit point finir! Et d'un autre côté, quels étranges abus, s'il n'y avoit pas quelque amertume attachée à ce dernier instant. On y courroit; et le plus léger chagrin nous armeroit contre nous-mêmes.

C'est même peu de ne pas craindre la mort, il est des occasions où il faut savoir la mépriser? Qui ne craint pas de mourir, est tout-puissant. L'Empereur Vespasien manda à Priscus, sénateur Romain, de ne point se trouver au sénat, ou, s'il s'y trouvoit, de ne pas dire son avis, et le menaca de le punir de mort. "Vous ai-je dit, réplique-t-il, que j'étois immortel? Vous ferez ce que vous voudrez, et moi je ferai ce que je devrai."

Connoître la mort sans la craindre, c'est le plus haut degré de perfection, où l'esprit humain soit capable de parvenir; c'est le plus haut point de félicité dont il soit susceptible de jouir dans cette vallée de misères; et il n'y a que le vrai Chrétien qui soit capable d'avoir cette fermeté. Je dis connoître la mort sans la craindre; et c'est dans l'union de ces deux choses, que se trouve cet effort de génie si digne d'émulation, et cette perfection de félicité, si capable d'embraser nos recherches et nos désirs; car, braver la mort sans la connoître; fermer les yeux à ce qu'elle a des hideux, pour la combattre avec succès; bien loin que ce soit une disposition éminente, c'est un déréglement d'honneur, c'est le comble de la misère.

On a vu des philosophes s'affranchir des frayeurs de la mort, mais ils ne la connoissoient point. Ils se la figuroient, on anéantissant l'homme, ou le faisant comparoître devant des tribunaux chimériques, ou suivie de quelque félicité imaginaire. Mais le Chrétien seul peut connoître la mort sans la craindre; lui seul peut mourir sans regret, sans remords, sans crainte. La religion le tranquillise: il espère un bonheur éternel, parce qu'il a vécu et qu'il meurt en homme de bien, en homme qui désire de voir Dieu et de le posséder à jamais.

Un philosophe du siècle, quelque force d'âme qu'il ait, quelque persuadé qu'il soit de l'anéantissement de son âme, sera agité de mille craintes à l'article de la mort. L'orgueil pourra l'empêcher de les faire connoître à ceux qui l'entoureront, dans ce dernier instant; mais sa conscience ne sera pas moins inquiétée. Il faut mourir Chrétien, pour mourir tranquillement; ou du moins il faut avoir une religion qui nous promette des récompenses après cette vie. Le néant est affreux, même aux méchans.

A translation from the French of J. B. H. De Saint Pierre, the celebrated authour of "THE STUDIES AND HARMONIES OF NATURE, *&c.*"

BENEFICENCE.

IF I had possessed ever so inconsiderable a portion of riches, I should have wished to procure for myself a thousand new pleasures. Paris would have become to me another Memphis. It vast population being unknown to me, I should have had a little chamber in its suburbs near the quarries; another at the opposite extremity on the banks of the Seine in a house shaded by willows and poplars; another in one of its most public streets; a fourth, in a house, surrounded by apricot-trees, fig-trees, cabbages and lettuces; a fifth, at the entrance of the city, with a vine-dresser and others.

It is doubtless easy to find lodgings of this description every where for a trifling consideration; but it is not so easy to find landlords and neighbours of virtuous dispositions. There is much depravity among the lower classes; but there are various means to discover good people. Employing these, I commence the search for my pleasures. A new Diogenes, I go in quest of men. As I seek out only the distressed, I have no need of a lantern. I rise at the break of day, and go to matins in a church yet only half visible. I find there poor workmen who come to supplicate the blessing of God on their daily toil. Piety which regards not human observation, is a certain proof of integrity; the love of labour is another. I have noticed, in rainy and cold weather, a whole family bent to the ground, weeding the herbs of a garden; here also are worthy people. Even the shades of night cannot conceal the appearance of virtue: Towards midnight, the glimmer of a lamp announces to me, through a garret window, some poor widow who has prolonged her evening to provide by her labour for her infants who are sleeping near her. These shall be my neighbours and the furnishers of my lodgings. I announce myself as a passenger, as a stranger who is seeking a spot for residence in that quarter. I intreat them to let me a part of their habitation, or to assist me in procuring lodgings among their neighbours. I offer a considerable rent;—and now I am in possession.

To attach these honest people to myself, I take care not to give money or bestow charity. I have more honourable means to gain their friendship. I employ them to provide for me a superfluity of provisions, by which they derive a profit. I reward their children for the little services which they render me. On a festival, I take the whole family into the country to dine with me upon the grass; in the evening the father and mother return to the city, well refreshed and stored with victuals for the remainder of the week. At the commencement of winter I clothe their children in

woollen; and their little limbs, warmed with my liberality, bless me, because my favours have not by an imaginary splendour frozen their hearts. It was the godfather of their little brother who made them a present of their clothes. The less we attempt to confine and rivet the obligations of gratitude, the more closely of their own accord will they bind themselves.

I have not merely the pleasure of doing good and of doing it seasonably, I have also that of amusing and instructing myself. We admire, in our books, the work of artists; but our books steal away half of our pleasures and of the gratitude which we owe them. They separate us from the people; and deceive us, by shewing us the arts with a great apparatus and by deceptive lights, like the representations of the theatre or the magic lantern. Besides, there is more knowledge in the head of the artist than in his art; and more intelligence in his hands than in the language of the writer who describes it. The objects of themselves furnish appropriate expressions; *Rem verba sequuntur.* A member of the commonalty has, in addition to this, a mode of observation and decision which is not unimportant: Whilst the philosopher rises as far as he can into the clouds, the plebeian stations himself at the bottom of the valley and distinctly views the world under different aspects. Misfortune forms him in time like other men. The propriety of his language increases with his years; and, I have often remarked that there is very little difference, in accuracy, perspicuity, and simplicity, between the expressions of an aged countryman and those of an aged courtier. Time effaces from their language and their manners that rusticity and overstrained politeness which society had introduced. Old age, like infancy, places all men on a level, and restores them to nature.

But some one may ask, "Is there nothing to fear in this mode of life?" May the conclusion of my days be found in the practice of virtue!

The Rev. W. Vint's Academy, Idle. J. VINT.

SOMNIUM SCIPIONIS.

Cum in Africam venissem, M. Manilio consuli ad quartam legionem tribunus (ut scitis) militum; nihil mihi potius fuit, quam ut Masinissam convenirem, regem familiæ nostræ justis de causis amicissimum. Ad quem ut veni, complexus me senex collacrymavit, aliquantóque post suspexit in cælum: et grates, inquit, tibi ago, summe Sol, vobisque reliqui cælites, quod ante quam ex hac vita migro, conspicio in meo regno, et his tectis P. Cornelium Scipionem, cujus ego nomine ipso recreor: ita nunquam ex animo meo discedit illius optimi atque invictissimi viri memoria. Deinde ego illum de suo regno; ille me de nostra republica percontatus est: multisque verbis ultro citroque habitis, ille nobis consumptus est dies. Post autem regio apparatu accepti, sermonem in multam noctem produximus, cum senex nihil nisi de Africano loqueretur, omniaque ejus non facta solum, sed etiam dicta meminisset. Deinde, ut cubitum discessimus, me et fessum de via, et qui ad multam noctem vigilassem, arctior, quam solebat, somnus complexus est. Hic mihi (credo equidem ex hoc, quod eramus locuti: fit enim fere, ut cogitationes sermonesque nostri pariant aliquid in somno tale, quale de Homero scribit Ennius de quo videlicet sæpissime vigilans solebat cogitare, et loqui) Africanus se ostendit ea forma, quæ mihi ex imagine ejus, quam ex ipso, erat notior. Quem ut agnovi, equidem cohorrui. Sed ille, Ades, inquit, animo, et omitte timorem, Scipio, et quæ dicam trade memoriæ.

Videsne illam urbem, quæ parere populo Romano coacta per me renovat pristina bella, nec potest quiescere? (ostendebat autem Carthaginem de excelso, et pleno stellarum, illustri et claro quodam loco) ad quam tu oppugnandam nunc venis pene miles? hanc hoc biennio consul evertes, eritque cognomen id tibi per te partum, quod habes adhuc à nobis hereditarium. Cum autem Carthaginem deleveris, triumphum egeris, censorque fueris, et obieris legatus Egyptum, Syriam, Asiam,

Græciam, diligere iterum consul absens, bellumque maximum conficies, Numantinum exscindes. Sed cum eris curru Capitolium invectus, offendes rempublicam perturbatam consiliis nequitiis mei. Hic tu, Africane, ostendas oportebit patriæ lumen animi, ingenii, consiliique tui. Sed ejus temporis ancipitem video quasi fatorum viam. Nam cum ætas tua septenos octies solis anfractus reditusque converterit, duoque hi numeri, quorum uterque plenus, alter altera de causa, habetur, circuitu naturali summam tibi fatalem confecerint: in te unum, atque in tuum nomen, se tota convertet civitas; te senatus, te omnes boni, te socii, te Latini intuebuntur: tu eris unus, in quo nitatur civitatis salus; ac ne multa, dictator rempublicam constituas oportet, si impias propinquorum manus effugeris. Hic cum exclamasset Lælius, ingemuissentque cæteri vehementius; leniter arridens Scipio, Quæso, inquit, ne me è somno excitetis, et parum rebus: audite cætera.

Sed quò sis, Africane, alacrior ad tutandum rempublicam, sic habeto: omnibus qui patriam conservarint, adjuverint, axerint, certum esse in cælo ac definitum locum, ubi beati ævo sempiterno fruantur. Nihil est enim illi principi Deo, qui omnem hunc mundum regit, quod quidem in terris fiat, acceptius, quam concilia cœtusque hominum, jure sociati, quæ civitates appellantur: harum rectores et conservatores hinc profecti, huc revertuntur. Hic ego, etsi eram perterritus, non tam metu mortis, quam insidiarum à meis, quæsivi tamen, viveretne ipse et Paulus pater, et alii, quos nos extinctos arbitraremur. Immo vero, inquit, ii vivunt, qui ex corporum vinculis, tanquam è carcere, evolaverunt: vestra vero, quæ dicitur vita, mors est. Quin tu aspicias ad te venientem Paulum patrem. Quem ut vidi, equidem vim lachrymarum profudi. Ille autem me complexus, atque osculans, flere prohibebat. Atque ego ut primum fletu represso loqui posse cœpi, quæso, inquam, pater sanctissime atque optime, quoniam hæc est vita, (ut Africanum audio dicere) quid moror in terris? quin huc ad vos venire propero? Non est ita, inquit ille. Nisi enim Deus is, cujus hoc templum est,

omne quod conspicis, istis te corporis custodiis liberaverit, huc tibi aditus patere non potest. Homines enim sunt hac lege generati qui tuerentur illum globum, quem in hoc templo medium vides, quæ terra dicitur: hisque animus datus est ex illis sempiternis ignibus, quæ sidera et stellas vocatis: quæ globosæ, et rotundæ, divinis animatæ mentibus, circulos suos orbesque conficiunt celeritate mirabili. Quare et tibi, Publi, et piis omnibus retinendus est animus in custodia corporis: nec injussu ejus, à quo ille est vobis datus, ex hominum vita migrandum est ne munus humanum assignatum à Deo defugisse videamini. Sed sic, Scipio, ut avus hic tuus, ut ego, qui te genui, justitiam cole, et pietatem: quæ, cum sit magna in parentibus et propinquis, tum in patria maxima est. Ea vita, via est in cælum, et in hunc cœtum eorum, qui jam vixerunt, et corpore laxati, illum incolunt locum, quem vides. Erat autem is splendissimo candore inter flammas circus elucens, quem vos, ut à Graiis accepistis, orbem lacteum nuncupatis: ex quo omnia mihi contemplanti, præclara cætera et mirabilia videbantur. Erant autem eæ stellæ, quas nunquam ex hoc loco vidimus: et eæ magnitudines omnium, quas esse nunquam suspicati sumus: ex quibus erat illa minima, quæ ultima cælo, citima terris, luce lucebat aliena. Stellarum autem globi terræ magnitudinem facile vincebant. Jam ipsa terra ità mihi parva visa est, ut me imperii nostri, quo quasi punctum ejus attingimus, pœniteret.

Quam cum magis intuerer, quæso, inquit Africanus, quousque humi defixa tua mens erit? nonne adspicis, quæ in templa veneris? novem tibi orbibus, vel potius globis, connexa sunt omnia: quorum unus est cælestis, extimus, qui reliquos omnes complectitur, summus ipse Deus, arcens et continens cæteros; in quo infixi sunt illi, qui volvuntur, stellarum cursus sempiterni: cui subjecti sunt septem, qui versantur retro contrario motu, atque cælum: ex quibus unum globum possidet illa, quam in terris Saturniam nominant. Deinde est hominum generi prosperus et salutaris ille fulgor, qui dicitur Jovis.

A TRANSLATION FROM THE LATIN OF CICERO,

IN HIS TREATISE DE SENECTUTE.

The young reader is requested to observe, that, in this fine extract from Cicero, is included the greatest expression of certainty respecting the immortality of the human soul to which the reason of man unassisted by revelation ever yet attained. Many of the arguments in it are what the men of the world commonly call "good natural reasons;" and the conclusions have been, to the enlightened philosophers of succeeding ages, matter of agreeable astonishment and surprise. But, after all, how full of wavering is this boasted persuasion! How unlike the 'life and immortality which are brought to light through the gospel!' *Beholding the errors of heathenism in its brightest form, may we learn to be yet more abundantly thankful for that grand dispensation of divine mercy and goodness,* 'the glorious gospel of the blessed God!' ED.

ON THE

IMMORTALITY OF THE SOUL.

I AM of opinion, P. Scipio and C. Lœlius, that your fathers, men very illustrious and most friendly in their intercourse with me, are still alive; and that their life is that life alone which is worthy of the name. For whilst we are shut up in these corporeal tenements, we have to perform various necessary duties and painful labours. The celestial spirit is borne down from its lofty abode, and buried, as it were, in the earth, which is a place opposed to its divine nature and eternal existence. But I believe that providence has formed the bodies of men with souls, that there should exist beings who might survey the earth, and who, on contemplating the order of the celestial phenomena, might imitate that order in the regularity of their mode of life. Not only have reason and discussion produced this belief in me, but the dignity and authority of the most eminent philosophers. I have heard that Pythagoras, and the Pythagoreans, almost of the same soil with us, who were formerly called THE ITALIAN PHILOSOPHERS, never entertained a doubt that we

have souls which emanate from the all-pervading divine intelligence. Besides, those things have been communicated to me which Socrates, on the last day of his life, in an eloquent discourse, advanced on the immortality of the soul; and he was pronounced by the oracle of Apollo to be the wisest of men. Why should I enlarge? Such is my persuasion, such are the sentiments I maintain,—since in the mind there is so great a rapidity of thought, such a tenacious recollection of the past, such a foresight of the future, and the principles of so many arts, so many sciences, and so many inventions that the nature which contains them cannot be mortal: And since the mind is always in a state of activity, and has no external principle of motion, because it moves itself; and will have no termination of its activity, because it never abandons itself: And since the nature of the soul is simple, and has nothing blended with its essence unlike or dissimilar to itself, it cannot be divisible; and if not, it cannot perish.

According Xenophon, Cyrus the elder, on his death-bed, said these things: "Do not suppose, my dear "children, that when I leave you I shall occupy no "place or shall entirely cease to exist. Even whilst I "was with you, my soul was not an object of sight; "but you perceived that it was in this body from those "things which I did. Believe it therefore still to exist, "even if you should see nothing."—The honours of illustrious men would not remain after death, if their souls were incapable of any effort by which we might continue to hold them in remembrance. Indeed I never can be persuaded that the soul lives whilst it is in a mortal body, and dies when it departs; or that it becomes incapable of perception, when dislodged from a body formed of brute matter: but rather I conclude, that when the soul, freed from all connection with the body, has become holy and perfect, it then becomes truly wise. Even when the nature of man is dissolved by death, it is evident whither every other thing departs

and to what it is reduced; for they all return to the place whence they originated; but the soul alone is invisible, both when it is present and when it departs. You now see, that nothing is so like death as sleep. But the minds of those who are asleep, clearly indicate their divine origin; for when under no restraint, they foresee many future things: from which it may be collected what they will be when completely emancipated from the fetters of the body.

Do you suppose (to boast a little of myself like an old man) do you suppose that I should voluntarily have sustained such hardships, day and night, at home and abroad in the army, if the same limits were to bound my glory and my life? Would it not have been much better to have enjoyed a retired and quiet life, without any labour or struggle? But the soul, elevating itself I know not how, always looked forward to futurity, as if when it should have escaped out of this life it would then at length have entered on a proper course of existence. Were not this a truth, the soul of every good man would not so ardently aspire after immortal glory. Does not every wise man die with entire resignation, and the man without wisdom with the greatest impatience? Does not that mind which has clear and extensive views appear to you to have the prospect of a better state; but a clouded and besotted understanding to have no such prospect? Indeed I am borne away with the eagerness of desire to behold your fathers, whom I respected and loved. Nor do I wish to meet those only whom I have known, but also those of whom I have heard and read, and of whom I have written. None, I am certain, should easily draw me back when on the way to their abode, nor attempt to form me anew as if I were another Pelias. Should any god offer me the privilege of becoming a child again, and to cry in the cradle, I should decidedly reject the offer. Indeed I should not wish to commence running a second time, after having once completed the course

and reached the goal. For what comforts has this life? Or rather, what distresses has it not? Life indeed may have its enjoyments, yet certainly they are attended either with satiety or limitations. But it is not in my inclination to mourn over life, as many and even learned men have done. I do not regret having lived, since I have so lived that I do not think that I was born in vain. I shall remove from this life as from an Inn, not as from my home. For nature has given us here only a place of temporary entertainment, not a permanent habitation.

O glorious day! when I shall go to the divine assembly and concourse of souls! when I shall pass from the rabble and dregs of this world! For I shall not only go to those persons whom I have already mentioned, but also to my Cato, than whom no better man was ever born, none more distinguished for piety; whose body I placed on the funeral pile, when he should have rather performed this office for me. But his soul, not deserting me, but affectionately looking back on me, departed into those regions where he judged that I should arrive:—Which misfortune I appeared to bear patiently; not because I did not feel, but I consoled myself with the thought that there would be no long separation between us.

These considerations, Scipio, (for you have told me that the subject has usually excited both your admiration and that of Lœlius,) these considerations make old age seem a light thing; not only not troublesome, but even pleasant. If I err because I believe that the souls of men are immortal, I err voluntarily; nor do I wish that this error in which I take pleasure, should be wrested from me whilst I live: And if (as some petty philosophers think,) when I am dead, I shall have no perception. I need not however fear lest the dead philosophers should make a jest of this mistake of mine.

Still if we are not immortal, it is a desirable thing that man should finish life at the stated time: For as

nature has fixed limits for all other things, so also for life. Old age is the closing of life, as of a tale that is told, the tiresomeness of which we ought of choice to avoid, especially when attended with satiety. I had these things to say concerning OLD AGE, to which I wish that you may arrive, that the things which you have heard from me you may prove by your own experience. W. VINT.

We received other good themes and translations from Master Washington, Watson, Morton, Neck, Gallini, Hill, Lewis, Roper, Levy, and Taylor.

Dulces ante omnia Musæ. VIRG.

Lines written at the Conclusion of Winter.

STERN winter now deforms the rolling year,
And blasts the beauties of the rising day;
While storms and darkness on the skies appear,
And ice-drops sparkle on the pendant spray:
Creation seems to slumber in repose,
Wrapt in a mantle of far-drifting snows.

The silent songsters of the shady grove
No more with joy their grateful praises sing,
In strains replete with harmony and love,
To greet the coming of the new-born spring;
Save where the redbreast seeks a scanty store,
And mournful does the changed scene deplore.

Now let kind pity prompt the generous breast
To sooth the ills of poverty and grief,
To succour those with want and pain distrest,
Whilst they implore compassion and relief:
For what were man, if heaven refus'd to spare?
What could avail his wishes or his prayer?

But see the soaring lark aspiring high
To greet the coming of the rosy morn;
With golden tints is deck'd the eastern sky,
And the clear dew-drop glitters on the thorn:
And soon shall beauteous spring with radiant smile,
Once more revisit Britain's happy isle.
Nile-Street, April 1st, 1816. J. DARBY.

SPRING.

No more the ice the purling stream detains,
Or earth is bound in adamantine chains;
No more rude tempests dim the eye of day,
Or gloomy clouds obstruct his cheering ray;
No more cold nightly exhalations rise,
Or fiery vapours glide along the skies.
Stern winter now has spent his utmost rage,
And blooming spring succeeds the hoary sage.

But hark! the cock proclaims th' approaching morn,
And ruddy streaks the eastern skies adorn;
The soaring lark through ether wings his way,
And chants his matins to the god of day.
Rous'd by his voice the feather'd tribes awake,
They leave their nests,—their nightly haunts forsake;
Each little warbler tunes the vocal wire,
And joins symphonious in the general choir.
Superior heard o'er the contending throng,
The thrush and blackbird chaunt their simple song;
The linnet, bullfinch, and the nightingale,
With warbling echoes fill the flow'ry vale.
Nay every bird, on every spray, does sing
A cheerful welcome to returning spring.

Behold what beauties burst upon the view!
The blasted fields contract a verdant hue;
The embryo leaves, unfolding by degrees,
In full luxuriance deck the naked trees;

From every clod fair vegetation springs,
And health comes floating on the zephyr's wings.
The finish'd garden blooms with matchless pride,
Fair lovely flow'rets spring on every side,
Drest in gay hues, magnificently grand,
The sport, the fancy of great nature's hand.
Where'er I turn my captivated eyes,
I see new beauties, see new charms arise;
Here blooms a rose—a gaudy tulip there,
Or this, or that, oh how divinely fair!
That not confin'd to mere exterior dress
Doth far more lasting, pleasing charms possess;
From its fair form ambrosial sweets exhale,
Perfume the air, and scent the western gale.
E'en when its transient short-liv'd bloom is o'er,
And its pale corse can please the eye no more,
It still emits (an emblem of the just,)
A potent fragrance from the blended dust.
Not so the tulip, (tho' its beauties shine,
Its lovely hues bespeak a hand divine,)
For all these charms are mere external show,—
(A striking emblem of the empty beau,)
No sweet perfumes attend its early bloom,
No pleasing odours waft it to the tomb;
Its beauty dies, and while its charms decay,
It joins unheeded with its native clay.
Ye race of fools, by base ambition fir'd,
Who feel most happy when you're most admir'd,
Oh take an int'rest in the tulip's fate,
And purchase wisdom ere it be too late.
Burn to improve, and strive yourselves to know,
And place just value upon things below.
For what is fame? What's honour, but a dream,
A bubble dancing on a rapid stream?
All earthly pomp and titles melt away,
As wax dissolves before the solar ray;
But white-rob'd virtue,—bright seraphic guest,
Nor age can wrinkle, nor can time molest;

Her lasting charms nor time nor death can move,
They bloom perennial like the pow'rs above.
Tho' earth should tremble, which so long has stood,
Suns turn to darkness, and the moon to blood,
Yet would fair virtue with composure stand,
And see worlds vanish at the Lord's command.
She'd flee to scenes of never-fading bliss,
Extatic pleasure, purest happiness,
Triumphant land on Canaan's happy shore,
Where one unbounded spring doth reign for evermore.

Leeds. TEASDALE BELL.

On the remarkable Properties of some Species of Plants.

THOUGH the line of distinction, between the animal and vegetable world, is in general sufficiently distinct, there are, in some plants, appearances of sensation, and even something which seems to approximate to what is called instinct. The sensitive plant, (MIMOSA SENSITIVA) when slightly touched, shrinks from the hand; if a stronger stroke be given to it, the leaves on the whole branch contract. The same irritability is observed in the Venus's Fly Trap. (DIONÆA MUSCIPULA.) The leaves of this plant are armed with spines, which spread themselves upon the ground, round the stem of the plant, and immediately and suddenly close upon any insect that may chance to creep upon them.

Light powerfully affects plants. The folding of the leaves and petals, in an evening and in the night, which is observable in many vegetables, is a circumstance most extraordinary. The contraction of the foliage of some species is so particular as to destroy their identity; especially when they are young, as I have observed in some exotic species of the *Solanum*. This wonderful property is not confined to plants kept under glasses; so that it does not appear to depend on heat. Indeed, in some instances, we see this sleep of

plants at noon-day; which proves that it is not altogether to the absence of light to which we must attribute this peculiarity. Heat is said to relax the tone of the vegetable organs; and it seems reasonable to conclude, that the fluids which the fibres contain, will be in some degree evaporated: a consequent weakness, or a slight flexibility in the part of the plant, so exposed to the heat, will, from well-known principles, incline it to the object from which the heat and light proceed. The leaves of the vine will turn even to the light, when separated from the branch, if they be carefully suspended by a small thread of silk or cotton; and the Cranesbill, (GERANIUM) if confined for some days in a dark room, will turn its branches to any crevice, or aperture, near to it, where a little light may chance to penetrate; and will even turn its foliage to the light emitted from a candle. The leaves of the HEDYSARUM-GYRANS are in constant motion during the day; and this motion is increased by the plant being placed in more light, and is retarded proportionably by different degrees of darkness. It is said indeed, that its shaking is considerably diminished, by the beams of the sun being hidden by a cloud.

There is a most remarkable peculiarity in the Scarlet-flowering Bean, generally called *Scarlet-Runners*. This plant seems to have a most positive tendency to twist its tendrils round its supporter, from west to east; while those of the Hop (HUMULUS,) as constantly turn in the contrary direction. The writer of these observations has repeatedly endeavoured to make the *Scarlet-Runners* wind with the sun, or from east to west; but no art which he has used could accomplish his design. The tendrils of the vine, (VITIS,) the Cucumber, (CUCUMIS) and the the Great Bind Weed (CONVOLVULUS SEPIUM) all turn in one particular direction. Whether this arises from a sort of instinct, or from some inherent property attached to the seed, by the Omnipotent Creator of all things, seems to be a subject

which is not to be comprehended by the finite capacity of man.

It would be no difficult task to multiply observations on this subject, as numerous indigenous plants exhibit properties equally suprising as those already noticed. The delightful science of botany is doubtless cultivated by many of the scientific gentlemen, whose lucubrations in the Leeds Correspondent shed honour on their names, and delight the numerous readers of that ably-conducted miscellany. It is hoped therefore, that some gentleman skilled in "Botanic Lore" will offer his observations on the subject, of which the writer has here attempted an outline. Botany is an elegant amusement, and a science which is of real use to mankind. To unfold the mysteries of nature, and to develop the economy established in the animal and vegetable world, are enquiries calculated to raise the affections of man, to the Authour of those objects which may engage his attention, or which may be the subjects of his speculations. Nature teems with wonders; but it must be confessed, that in our attempts to investigate its phenomena, our little taper of intelligence is often too dim to dispel the darkness which envelopes us; and in accounting for them independently of the First Great Cause, our minds are left in a state of continual oscillation and uncertainty.

Thorp Arch, March, 1816. I. F.

The extract communicated by PRINCIPIA, on the difference between Experience and wisdom, continued from page 21.

On Geometry and Mechanism in Nature.

THE second medium leading to wisdom, by which the arcana of invisible nature may be unlocked and revealed, is Geometry and rational philosophy; by means of which we are enabled to compare our experiments, to digest them analytically, to reduce them to laws, rules,

and analogies, and thence to arrive at some more remote principle or fact which before was unknown.

Mere experience or experiment is incapable of unfolding any thing and of reducing it to its more simple parts. It cannot so arrange facts that bear a resemblance to each other, as to discover what was unknown by observing its similarity to things that are known; for this is the office of reason. To retain a great abundance of observations in the memory, and afterwards to form conjectures or conclusions respecting things unknown from their similitudes and analogies to such as are known, and thus to ground our discourse upon a connected chain of experiments, is also a familiar and natural method of attaining wisdom.

The whole world itself, as consisting of the elements of the mineral and vegetable kingdoms, is a pure system of mechanism; and so is the animal kingdom too, as to its anatomical organization. The science of mechanics itself, with all its powers, geometry with all its figures and numbers, and philosophy with all its comparative and proportional reasoning, sprang solely from the elementary world; they are the offspring of the elements of which they were conceived and born. The science of mechanics is the law of nature herself, as she acts and moves in the elements; and it is according to this that her parts have their motion both simple and compounded; without the elements and their regular disposition and motion, no mechanism could exist. As therefore the science of mechanics is the law of elementary nature, it follows that the world itself is governed by suitable laws and rules, and that the whole is a grand piece of mechanism: and this becomes more evident when we observe that nothing can be set in motion without being governed by some mechanical law. If motion be supposed, the figure of that motion must be supposed, and also its space: consequently, if there be figure and space, as well as motion, the whole is something mechanical, and is capable of being redu-

ced to the laws of geometry. The very attributes of motion, without which it cannot exist, are geometrical, being figure and space. When we form an idea of any body, however small, as soon as we consider it as being limited we regard it as something geometrical, because it possesses figure and quantity acording to its peculiar dimensions. It may also be considered as subject to the laws of proportion in itself, because it possesses distance between its limits; and between one point of that distance and another there exists proportion: the case is the same in other instances.

Thus not only motion, but every finite thing in a state of rest, possesses attributes which are purely geometrical. Geometry, therefore, accompanies the world from its first origin, or first boundary, to its last, and is inseparable from it: and so also do the principles of mechanics, though they might be different in a world differently formed and in elements differently formed and arranged. And thus, although there may be innumerable finite worlds, nothing can exist in any of them which does not depend upon some mechanical principle: and a like principle of geometry must be common to them all. Whoever supposes the world to be any otherwise constituted, takes refuge in obscurity to conceal his ignorance, and to preserve his reputation as a philosopher in the republic of letters. But every well-formed mind must admit that the world is composed of elements; that elements are composed of particles; that particles are composed of spaces and figures; that figured particles are the result of motion, and of situation or disposition suited to such motion; and that motion and situation have their proportions. As all things in the world are mechanical, and possess motion and limits, it also follows that the smallest natural things, as well as the greatest, follow a mechanical order, and that the minutest and largest objects are governed by similar mechanical principles. And though the particles of the elements are invisible, and in a great mea-

sure elude the observation of our senses, yet as they are fluid and limited, they are geometrical, and must flow and subsist in a mechanical manner. The case herein must be the same both with the objects that are within, and with those that are above, the sphere of our sight. That the equilibrium and motion of the greater bodies follow the common and known laws of mechanics, is certain; we see it to be the case in the vortex or circumvolution of our sun, in the planets, in the earth, in the satellites that revolve within the boundaries of a greater vortex or circumvolution, and move through their proper orbits, with perfect regularity, in the figure of an ellipsis, exactly as smaller bodies would do if they were made to revolve in a similar figure. These immense bodies are governed by the same law, or the same centripetal and centrifugal tendency, which is observable in small bodies that are in like manner to revolve round their centre. A similar proof is also afforded by the animal kingdom, in which we see the motions of whales, elephants, and other animals, effected by means of tendons, nerves, muscles, and fibres, which move their feet, legs, toes, and all the other members of the body.

In all animals we see the blood and fluids flow and return through the large and small arteries and veins, and through their proper ducts and vessels: and either, as in vegetables, proceed to certain fixed boundaries, or continually retrace their steps and perform the same course over again. And we also see how the lungs perform their alternate motions, like a pair of bellows, according to the influx and reflux of air. That all this is mechanically effected, we have ocular testimony, as the nerves, fibres, and muscles, all properly shaped and adapted to their respective motions, lie open to our view; we are thus enabled to investigate these primary mechanical and hydraulic machines, to handle as it were these original motive powers, and to demonstrate that they all depend upon mechanical principles. The

same observation is true of the organs of the senses. For it is known that the undulating air flows into the ear, and occasions in its coat or *tympanum* a motion imitative of itself, and that it again continues the same motion through its passages, coats, *tympana, cochleæ, mallei, incudes,* and the other instruments of sound, towards the interior parts; so that the undulation of the air seems to have formed a mechanism of its own, that it may be received and transferred farther towards the membranes of the same kind that lie within, for the reception of sensation. What a wonderful mechanism is apparent in the eye! where there are so many coats, so many humours and fibrils, and nerves leading from them, towards the interior parts; by means of which whatever is received from the ether in the eye, insinuates and propagates itself from thence towards the coats of the same kind in the meninges, and thus deeper and deeper: so that the ether seems to have formed a mechanism of its own in the eye, by which its undulations may be received, and be farther transferred towards the interior parts till sensation is experienced. These contrivances and minute machines, most exactly formed, according to the laws of mechanics, for the reception of the modifications of the air and ether, it is in our power to view, examine, and scrutinize in all their parts, and to see how their members and coats are produced from the interior recesses of the head into the light of day, that the elements may be able to operate immediately upon them, and more speedily convey the motions impressed on them from thence towards the interiors, which is effected by degrees, by first affecting the coats of the same kind, and then such as are smaller and possessed of more acute sensibility.

From these observations we may conclude, that the animal body is governed by mechanical principles; to which conclusion we may add, as will now be demonstrated, *that the same kind of mechanism is observable in the smallest animal, as in the largest.*

(To be continued.)

A BRIEF MEMOIR

OF THE

Life of the late Mr. John Ryley,

TEACHER OF THE MATHEMATICS IN LEEDS.

THE observation is very trite, though the truth of it *seems* to be confirmed by frequent repetition, that "the biography of a man of letters is devoid of interest, and that the perusal of it affords little amusement or instruction." This remark, viewed generally as it respects the taste of the multitude, is not without some foundation. But to those who are walking in the same path to honourable distinction, who have exerted similar powers of ingenuity, and exercised themselves in a course of rigorous self-denial and close application for the attainment of the same object, no circumstance relating to the triumphs of genius or the rewards of perseverance will appear trifling and uninstructive.

The acquisition of knowledge, in a young man especially, is a mere act of selfishness. He pursues it for his own gratification, that he may obtain for himself what he has not in possession and is conscious of needing. This is a powerful principle, implanted in the human breast for valuable purposes. When left to expend itself, it wearies out the person who is the subject of it, transforms his views, and works within him a willingness to communicate to all around him his acquired stores of instruction. The number is small of those who apply themselves diligently to any literary pursuit, purely with the intent of benefiting others: This is in general but a secondary consideration, prior to which is the craving desire of satisfying the native curiosity of the mind.

To philosophize on the phenomena and tendencies of the mental powers, would be nearly endless. It may not, however, be impertinent to say, that this selfish prin-

ciple is neither weakened nor destroyed when it seeks for notice and commendation; but rather, that it runs through another channel and in a different direction. The reputation after which a literary man aspires, is that of a benefactor. If ever his breast is animated with such a sacred passion as the wish of being esteemed by his learned contemporaries or future generations, it is that he may be noted by them as one that has achieved a bloodless triumph by contributing somewhat towards the general welfare of his species. The rescue of a whole nation from imminent public danger by the skill and bravery of a commander, makes a sensible impression on the multitude thus delivered. The state of security in which they are placed, when contrasted with their former hazardous circumstances, causes them to speak of their chief and his followers in terms of the highest admiration and applause. Their merits are conveyed to wondering posterity, stamped with the exaggerated yet grateful testimony of all whom they have preserved. The many are also fond of pomp and pageant show. Their sensations are gross, and must have something visible to excite them. To the inward and durable delights of the mind they are often strangers; and, where a taste for these is wanting, it is not wonderful that the brief detail of a learned man's pleasures and employments should appear insipid and uninteresting.

The inventor of an useful science, or one who clears away the rubbish which conceals the avenues of knowledge, is a greater character, and more worthy of a nation's gratitude, than the destroyer of thousands, or the conqueror of tens of thousands. But the individuals most unambitious of distinction are frequently those possessed of the greatest merits, of which every one acquainted with them appears sensible except themselves. It is not too much to say, that in no country in the world is such general public attention bestowed on learned men, and their labours, as in England. This, it is hoped, will always stand the proudest test

of our national glory, the pre-eminence of our people as a well-informed community. This induces us, instead of neglecting our lettered benefactors, to assist in drawing them out of obscurity and ranking them among our monuments of greatness. However wishful to decline, while living, the blushing honours that crowded thick upon them, the same objections are no longer in force when they die, as they have then no power to give utterance to them. Their survivors who loved them, have then an opportunity of celebrating their virtues, and bringing them to life again in the remembrance of their friends.

Few men in Yorkshire have been more extensively useful than Mr. JOHN RYLEY, in smoothing the path to the abstruse parts of the Mathematics. He was the eldest son of Mr. Samuel Ryley, a respectable farmer and clothier, of Old-Coats, near Pudsey, Yorkshire, where the subject of this memoir was born on the 30th day of November, in the year 1747. He received at an early age such a common education as the school of his native village afforded; and was afterwards employed at home, till he was upwards of twenty-one years of age, in the joint occupations of husbandman and manufacturer.

The cloth-manufacturers in the neighbourhood of Leeds, and in other parts of the West-Riding of Yorkshire, are generally either the owners or occupiers of small farms, the cultivation of which is a sweetly-varied employment to themselves and their families. Even in the present improved state of agriculture, to meet with spots of ground better managed than theirs would be difficult. Their homely but commodious house, in every corner of which may be found a loom or other implement of trade, is generally built in a field, or very near one, for the convenience of warping and other purposes of business. Early every morning the father, and those branches of his family that are capable, busy themselves in weaving such a portion of a *piece* of cloth as they had set themselves to finish, or as had been

appointed to them as their share of labour. In the afternoon when their allotted toil at the loom is ended, they usually apply themselves to something beneficial and necessary in the practice of husbandry. Before it is late in the evening, the whole family retires to rest, fatigued with the wholesome changes of exercise in the day. Industrious and frugal in their habits, they are generally accounted men of integrity and honour. Such a quantity of small freehold estates are in their possession, that their numbers have more than once turned the scale in a general election of a member for the county. No part of the community has suffered more severely than they from the rude shocks in trade, and commercial distress, with which at various periods during the last thirty years this country has been visited. When, however, credit has been again established, and commerce has resumed its former tone, no persons have sooner recovered from the concussion, and regained the position which they had previously occupied in the ranks of society. Their habits of labour and perseverance have contributed to this result.

Among this hale and hardy race of men, who in their double capacity of tradesmen and farmers possess many advantages, Mr. Ryley passed the years of his life prior to manhood; and laid the foundation of a sound constitution, which after a hard struggle it was scarcely in the power of his severe studies in future life to impair.

He had made himself acquainted with arithmetic at school; and afterwards at home perused some works on Mensuration: These disclosed new fields of science to his inquisitive mind. Having imbibed a taste for the lore contained in these and other works, he begun in his twenty-fourth year to devote all his hours of leisure to these pursuits. So assiduous and successful was his application, that, in the course of other four years, he was found to be sufficiently qualified for engaging with the Rev. Mr. Hutchinson as Mathematical Teacher at Drighlington-school. This was a situation

which required considerable mathematical attainments; and while he held it, which was not much more than a year, he acquitted himself with great credit, and gave full scope to the bent of his mind in rioting on all the luxuries in the new works with which he there became acquainted.

During his residence at Drighlington he first dived into the hidden recesses of abstract quantity. Mr. Crookes, of Leeds, was at that time the most famous Mathematician in this part of the country; and to him Mr. Ryley resorted for instruction in Fluxions and the higher branches of Algebra. His previous acquirements were of the greatest service to him, as he was by means of them the better qualified to derive profit from the directions of a living preceptor, a privilege which till now he had never enjoyed. He has been heard sometimes to relate, that while with Mr. Crookes, he had one day taken uncommon pains in solving a problem which related to the rectification of the circle, or of some other curve, in which he used the inverse method of Fluxions. He completed his undertaking, as he thought, in a masterly style: but on comparing his result with that of the book from which he had taken the example, he found his own to be erroneous. He communicated to Mr. Crookes the cause of his vexation; and so subtle and concealed was the single fault, that it occupied the undivided attention of his master during two days to discover in what step of the process the error lay. This anecdote is not mentioned as a proof of any uncommon precocity of talent in Mr. Ryley; because many younger men than he then was, have solved without a mistake problems equally difficult: It is rather introduced to shew that the master was competent to teach and persevering in his province, and that his scholar was no novice and not to be deterred by difficulties.

The tuition which he received while at Drighlington from Mr. Crookes, was at first only occasional; and that when he could spare half a day from his regular

scholastic avocations. But he afterwards went to Leeds, with an express view to his improvement in mathematical science under the Archimedes who resided there; and remained there upwards of a month, either at one of his half-yearly vacations or immediately after he had left Drighlington.

His fame as a proficient in high sciences being now established, his friends became very importunate with him to commence a school in his own neighbourhood. He yielded to their wishss, and opened a school in Pudsey, where he received a good share of that encouragement which his abilities entitled him to expect. Other motives besides the solicitations of relatives, induced him to enter on the profession of a teacher on his own account.—We find him this year, soon after his settlement at Pudsey, engaging more deeply in a pursuit, for precipitancy in which a young man in his twenty-ninth year cannot be blamed. This was his marriage with Miss Dawson, of Topliff, near Wakefield, an union which he had some time contemplated, and which, he found, the longer it was delayed would the more seriously retard his favourite studies; for, with a trifling substitution, it may be truly asserted,

Non bene conveniunt, nec in unâ sede locantur
Mathematica et Amor.

This happy event having taken place to his entire satisfaction, he betook himself again with diligence to his former course, having now for his encouragement, a home of his own and a cheerful wife, two *desiderata* which every man of sense desires as soon as possibleto obtain.

Having an eye to the better means of support for himself and family than he was ever likely to enjoy in Pudsey, after continuing there a year he became a candidate for an excellent situation, as schoolmaster at Beeston, which was then vacant. He succeeded in his application; and attained to what he then considered to be the boundary of his wishes,—an easy competency and the means of indulging in literary recreations. But that which in one period of life is accounted a suffi-

ciency, is, in the course of years and under change of circumstances, often found to be but a trifling remuneration for services performed. During the early part of the thirteen years that he remained at Beeston, his family was but small and his wants very circumscribed. But he began to be blessed with children, and every one brought with it an increase to the general wants of the family. These demands his income was fully adequate to satisfy; and so he would have always thought, but his friends who had been increasing in number, were continually looking out for some other place more suited to his talents, and affording him a more handsome salary. He however had no unreasonable desires to gratify; and never made himself uneasy about futurity.

While at Beeston he became a correspondent in that great encourager of youthful mathematicians, the Ladies' Diary. His contributions were not confined to one branch, but extended to the whole course of the Mathematics. He attempted to solve all the questions proposed in each number. The intelligent editor of that work soon distinguished him as a well-informed mathematician; and many a fairly-won prize has been adjudged him in that work for his superiority.

TO BE CONTINUED.

On those Risible Images to be seen in Minsters and ancient churches.

The Friars and Monks, so often confounded by modern writers, were two distinct orders. The Friars were first instituted in the 14th century, and were designed by their preaching to oppose the Lollards, who who were then gaining ground. They travelled through the country acting as priests and confessors, (while the monks were chiefly confined to their respective Monasteries,) to the great annoyance of the secular clergy, whose revenues and emoluments were considerably diminished. These, in revenge, erected those ludicrous figures of Friars and Nuns in their Churches.

QUERIES.

(1) *By* JOB SHAINEN.

It is said that the bulk of all substances, whether solid, liquid, or aeriform, is expanded by heat and contracted by cold. If this be true, why does water occupy more space when frozen into ice than in its liquid state?

(2) *By* ANASTHIMENES.

Is coal a vegetable substance or a semi-metal?

(3) *By the same.*

Why do we feel the heat of the sun through a window when the glass through which it passes remains cold?

(4) *By* MISS HODGSON, *York.*

Which is the strongest of the passions in the human mind?

(5) *By* GRAMMATICUS.

An expression equivalent to "he used" is often needed in composition. What phrase will be its most elegant substitute?

(6) *By* Mr. T. PILKINGTON, *Haslingden.*

By what means are flies enabled to walk the ceiling of a room, with their bodies downward, or the underside of globes which are suspended in many houses as ornaments?

(7) *By* INVESTIGATOR.

The contributors to the Leeds Correspondent are well acquainted with the peculiar style of Addison and that of Johnson. Will some of them have the goodness to shew in what the excellence of each style consists and on what subjects it can be most appropriately employed? Will they at the same time more distinctly point out this difference by composing an essay on any subject and clothing it in the characteristic language of each of these authors?

(8) *By* QUÆSITOR, *Manchester.*

Is the sun a great globe of fire? If not, what is that great luminary?

ANSWERS TO THE QUERIES

Which were proposed in the First Number of the Second Volume.

Answers to the Queries were severally received from Messrs. F. Charlton, J. Baines, Shenstone, H. Armitage, J. Hill, T. Pilkington, and from Anasthimenes, Quæsitor, Elbean, Grammaticus and Investigator.

(1) The two grand principles or faculties of the human mind.

By Mr. J. BAINES.

'There are three faculties belonging to the human mind, viz. the understanding, the imagination, and the memory.

The understanding is that power which receives ideas, attends to, compares, reasons, and judges on the different objects which come within the sphere of the human intellect. Sensations are also referred to this faculty, as being the original stamina of thought. Mr. Locke calls it *the faculty of thinking*. Its abilities, or different degrees of comprehension, as enumerated by Dr. Kirwan, are genius, penetration, solidity, depth, sagacity, subtlety, perspicuity, and power of illustration. It is by the aid of this great faculty of the mind that man has controuled and subdued, to his own use, animals which are vastly superior to him in strength, and has converted the productions of the earth to his own accommodation and enjoyment. It is this which promotes the arts and sciences, which contribute to ameliorate the condition of man. By discovering truth, it shews the necessity of religion, and makes man accountable to his Creator for his actions. It is therefore the motive for virtuous conduct, and also for the pursuit and attainment of permanent and substantial happinesss. On the cultivation of this faculty depends the chief superiority of one man above another; all other pretensions to eminence are absolutely false.

The imagination is the power of compounding, curtailing, and arranging ideas, and of combining them

in a manner or form more intelligent than that in which they were originally impressed. It enables man to admire and enjoy whatever is beautiful or sublime, in the productions of art, or the works of nature. The abilities which belong to it are, genius, or an enthusiastic power of fascinating by exciting the strongest emotions; also talent, wit, acuteness, and taste.

And lastly, the memory is the capacity of recovering, recollecting, or retaining these states, which are more or less enfeebled, by which the mind was antecedently modified, with an instinctive persuasion of their preceding existence, by which instruction and entertainment are deduced from the records of history and the review of past events. The abilities of the memory are, the facility of acquiring and committing to it the objects sought to be remembered, the power of retaining them for a considerable length of time, and the readiness of recollecting them.

From the above description of the three faculties, it may be fairly inferred, that THE UNDERSTANDING and THE IMAGINATION are the two grand principles or faculties of the human mind.

By Mr. T. PILKINGTON.

It appears that all the operations of the human mind are reducible to two grand principles—*the Will*, as the primitive principle of life, which is the seat of our love or affections, and the other, which is a kind of modulation of the former, is denominated *the understanding*, and we may view it in a secondary place or manner, as a proceeding-forth of the *Will*, its parent, and as a subordinate faculty, by and through which our designs are accomplished.

The will principle may be compared to a first mechanical power; but it is by means of well-constructed machinery, corresponding to the understanding, that the intended effects are ultimately produced.

And again the primary principle of life, which is love, and which is of the will, may be likened to heat

or caloric; for as caloric insinuates itself into the blaze of light thence originating, in like manner does the will insinuate or infuse itself into our understanding, producing in us the light of the mind, which light is of such a quality as is that of our will or affection. Such as is the quality of the heat, such will be the quality of the illumination produced; and such as is the quality of the will, such will be the quality of the understanding, because the end or design is manifested by it. It is upon this ground that the understanding may be called a modulation or proceeding-forth of the will, "for that which is of the will, forms itself, and renders itself visible in the understanding. They are distinct from each other, but so created that they may be one; and when they are one, they are called *mind*. Wherefore of these consists the human mind, and all the life of man is therein.

"Our love itself, and the affections that relate thereto, have their abode in the will; and science, intelligence and wisdom have their abode in the understanding. The will breathes its influence into the contents of the understanding, producing thereby favour and assent; hence it is that every man's true nature and quality depend on the nature and quality of his will. The will therefore is the essence of the life of man; but the understanding is the existence thence derived. And whereas essence is a mere nothing unless it exist in some particular form, so consequently will is a mere nothing unless it exist in the understanding; therefore the will maketh itself a form in the understanding, and by that means cometh to light and is made manifest. Love in the will operateth as an end; and in the understanding seeketh and findeth causes, whereby it may exert itself to produce effects. And whereas an end is a purpose which intendeth something, it is also the purpose of the will, and by means of intention entereth the understanding, and urgeth it to employ and consider of means, and determine on what may tend to produce effects."

Hence it appears that the will and understanding are conjoined like cause and effect; and also that things of the will are attributable to man, but not merely those of thought.

(2) The plural of "valley."

By Mr. J. SHENSTONE, *Plymouth.*

On the inspection of a few singular nouns, ending in y, such as *glory, beauty, fly, spy, delay, fray, alley;* we shall find that their plurals are generally written *glories, beauties, flies, spies, delays, frays, drays, days, alleys.* This rule may therefore be deduced, that if the y be preceded by a vowel, the plural is formed by the addition of s; but if by a consonant, the y, is changed into ies; consequently the plural of valley, is valleys. But all writers on grammar are not of this opinion; for some of them have made the plural of valley, vallies; yet have given no rule to shew why it should be so. They merely say, that "*nouns ending in y, form their plurals by changing it into ies, except*" naming a few words; to which a great number might be added, sufficient at least to give a more definite rule.

(3) The fertilizing properties of Snow.

By Mr. H. ARMITAGE.

Snow on account of its being a bad conductor of caloric, is wonderfully calculated to preserve that degree of congenial warmth which is necessary for the protection of the vegetable tribe: Its affinity for oxygen, gives it its fertilizing quality. The snow melting and penetrating into the earth, imparts oxygen to it, which promotes the germination of seed. This oxygen combines with the carbon of the earth, and forms carbonic acid, and thereby acquires more solubility; while the water contributes to excite that activity which had been rendered dormant in the roots on account of cold.

By ANASTHIMENES.

It has been erroneously believed that snow possesses more fertilizing properties than rain. But the real

difference has, by Marggraf, been ascertained to be exceedingly small. When abundant in quantity, it serves to defend corn and other vegetables from the severity of winter frosts; as it prevents the internal heat of the earth from being elicited through the surface, and consequently ameliorates the soil.—The plants being thus sheltered, shoot forth in the spring with renewed vigour; and being cherished by the genial rays of the sun, vegetate with increased luxuriance.

By Mr. J. BAINES.

The experience of all ages has taught mankind, that a great quantity of snow lying upon the earth in winter for a considerable time, has contributed, in some respect, to its fertilization. There is not a better covering provided by nature than snow, to secure the corn, plants, and trees from the pernicious effects of cold weather, and to shelter the earth from freezing winds. But to account for its fertilizing the ground, we must consider its formation as particles of vapours frozen together in the atmosphere. The shape of the desending flakes is that of little hexagonal stars with rays; which is probably owing to the saline particles that float in the air and unite with the snow or frozen vapours; the particles of salt serving as a kernel, and the vapours collecting round them, assume the form of a crystalization. Hence, when the snow is softened by the sun, or the temperate air dissolves it by degrees, those saline particles either enter the earth or lie upon its surface, and tend, by their fertilizing property, to invigorate the shoots of plants.

(4) On the petrifaction of the bones of animals.

By ANASTHIMENES.

When an animal or vegetable substance is buried under water or in wet earth, it is slowly and gradually decomposed: for as each successive particle of the animal or vegetable is destroyed, its place is supplied by a particle of siliceous earth conveyed thither by the water. In the

course of time it is entirely destroyed; but the silex has completely replaced it, having resumed its form and apparent texture, and appears as if the animal or vegetable itself were changed to stone. But it is impossible for either animal or vegetable substances to be converted into stone. Yet they may be reduced by decomposition to their constituent elements, but cannot be changed into elements which do not enter into their composition. Therefore no distinct species of "the bones of several animals," as intimated in the query, are liable more than others to be converted into stone.

By Mr. J. BAINES.

The transmutation of the skeletons of plants and animals, from the vegetable and animal kingdom into the mineral, is one of the most surprizing operations of nature. Fossil Crocodiles have been found near Honfleur, and in several other parts of France, and in different counties in England. These remains were generally imbedded in a matrix of hard limestone. The bones of a rhinoceros have been found in the neighbourhood of Canterbury; and some remains of the Hippopotamus have been dug up in Gloucestershire. In the year 1770, there was discovered, at Maestricht in the Netherlands, part of an enormous head of an animal in a solid stone, which, from every appearance, seemed to have been an inhabitant of the ocean. Stag's horns, of about five feet in length, have been dug up at Ballyward, near Bally Shannon, Twivy near Dublin; and at Portumery, in the county of Galway, Ireland. Extraordinary skeletons of Elephants have been found in America, Siberia, Sweden, Switzerland, Italy, France, Great Britian, and along the banks of the Rhine and Danube in Germany. A cursory inspection of the teeth is sufficient to determine that they are not those of the African Elephant, though they bear a close resemblance to the Hindostan species. M. Cuvier supposes, from a comparison of the bones of the present with those of the fossil Elephant, that those which have been found fossilized are of a species differing as widely from the

Asiatic Elephant, as the Horse does from the Ass; and therefore they must have belonged to a species which is at present entirely unknown. Several species of fishes have been found whole and perfect, even to the smallest scale; and also prodigious numbers of shell-fish, some of which are of a species that is not at present to be found alive. All sorts of petrified plants, and whole trees converted into stone, have been found in beds of soft earth.

To account for these great phenomena of nature, we may observe, that nothing will petrify in the open air, nor has a barren dry earth any petrifying quality; it is therefore probable that petrifaction is effected by soft moist earth which contains strong particles of a dissolving nature, capable of penetrating into the cavities of animal and vegeble skeletons, and of uniting with them as soon as the parts of the body itself evaporate, or are absorbed by alkaline substances. The famous dropping-well at Knaresborough in Yorkshire, possesses the property of petrifying in the most eminent degree. It drops from a porous rock; and the ground upon which it has fallen, is, to the extent of twelve yards, changed into solid stone. A small rivulet runs from this well and falls into the river Nid, where it has formed a rock that stretches several yards into the stream. A bird's nest being placed on the spot where the water falls, has been in a short time converted into stone.

(5) How ought we to parse these words, "The Lord, he is good?"

By GRAMMATICUS.

In our ancient phraseology many expressions occur which sound uncouthly when contrasted with modern refinements. The sentence proposed has a construction of this kind. Many others containing two apparent nominatives, are to be found in the best authors of the seventeenth century and in the common

English version of the scriptures. All of them admit of the same complete and easy solution, which is,—that a preposition preceding the first member of the sentence is generally to be understood, signifying *As to, As for, Concerning,* &c. The sentence will then read, "*As for* the Lord, he is good," which is an idiom very common with the Roman writers; and it is probably in imitation of them that the repetition was adopted by our ancestors.

Some persons have thought, that the former part of such a sentence may be correctly accounted for by calling it *an interjectional phrase;* and have represented it as a natural interruption occasioned by the mind of the speaker being so absorbed in his subject as to break off in the midst of what he intended to say, and attempting afterwards to continue it in a style somewhat altered, at once betokening the poverty of his language and its inadequacy to the importance and dignity of his matter.—But this latter method of accounting for it is not applicable in every case. Nor is the example itself, because of its tautology, deserving of imitation.

By Mr. J. HILL, *Kilham Academy.*

The construction of this sentence cannot possibly be made consonant with the rules of Syntax: for unless we can conceive of some word understood governing *Lord,* we shall have two nominatives to one verb, that is, the pronoun will be used in the same sentence as the noun for which it stands. It appears, however, from the Bible, in many parts of which is the same construction, that it is an emphatical mode of expression. The analysis of the sentence will be as follows:

The, the definite article.

Lord, a proper substantive, of the third person, singular number, masculine gender, and nominative case to the verb *is.*

He, a personal pronoun, and used in the same sentence with its noun for emphasis' sake.

Is, an auxiliary and neuter verb, in the indicative mood, present tense, third person, singular number, agreeing with its nominative word *Lord*.

Good, an adjective in the positive state, and belongs to its substantive *Lord*.

(6) On Oxygen as the acidifying principle.

By Mr. J. BAINES.

Oxygen is well known to be an acid body, and the basis of all acidity; but hydrogen is an alkaline substance, and when these two are mixed together in about the ratio of 85 parts of the former to 15 of the latter, they form water. The alkaline quality of the one destroys or neutralizes the acid of the other, and water is formed at the exact point of saturation.

(7) Why do drops of rain on the leaves of plants assume a globular form?

By Mr. J. BAINES.

The cause of the sphericity of bodies is the universal power of attraction which the Creator has impressed on all matter in the universe. All the particles which compose a drop of rain are attracted towards its centre, and being in a state of fluidity, they can only be at rest when every portion of its surface is at an equal distance from its centre, that is, when it becomes a perfect sphere. This is really the case with a drop of falling rain, but when it is deposited on the leaf of a plant, the lower surface is a little flattened, and the other part of the circumference extended, so as to form the greater segment of an oblate spheroid. The cause of this alteration is its adhering to a flat surface.

(8) On Gretna-Green marriages.

By FIDELIS.

On the validity or illegality of Gretna-Green marriages I pretend not dogmatically to pronounce. That is the province of lawyers. But when they occasionally disagree on a matter of this importance, it is not an

impropriety on the part of common sense to step in and shew an opinion. That two fugitives who, as it often happens, have scarcely ever seen each other, the one of them generally actuated by motives the most mercenary and unworthy of a man, the other the victim of vanity in giving unbounded credit to the assertions of the very first man that had an opportunity of telling her that he was about to die in consequence of his love for her,—that two such fugitives, by the mere act of setting foot for a couple of hours on Scottish ground, can be so far metamorphosed as to be accounted inhabitants, and entitled by a short stay to all the privileges of that country, is an opinion that outrages all decency, and is a complete burlesque on social usages. No country calling itself civilized, is so defective in its public institutions as to countenance such a perversion of them as this argues.

It is almost universally required that the parties desirous of marrying should, before that ceremony takes place, have been resident during a definite period in the town, or its vicinity, where the marriage is to be celebrated. And this is done that a correct judgment may be formed, by persons on the spot, of the legal disabilities or personal impediments that may exist in either of the parties prohibitory of their union.

On this general ground an action at law would lie against the parties; because, even according to the law of Scotland, the tagging together of two silly unknown runagates is illegal. And many of *the dear couples* would find it to be so to their sorrow, were it not that their parents or friends consider the legal remedy to be too slow for the disease. The illegality of their Scottish marriage would have to be proved in that country, before proceedings could be taken out against them in an English court of justice. And in that interval cohabitation, with all its consequences, could not be hindered. It is the knowledge of this circumstance that is the real cause why this unmanly practice is occasionally resorted to by sharpers or hypocrites with impunity.

By Mr. FOSTER CHARLTON.

The tribunals of England have always countenanced marriages contracted in conformity to the laws of the country where they had their origin. When the validity of a Scotch clandestine marriage is tried in England, it is left exclusively to the decision of the law of Scotland applicable to such a case; and this law never having resiled from the antient canon-law on which it was founded,—that mutual consent alone was sufficient to constitute a marriage,—it is obvious that private, irregular, or clandestine marriages are legal in Scotland; the ceremony is not at all necessary, and two persons, without the intervention either of a priest or any other individual, may contract a valid marriage. Gretna-Green (the far-famed temple of Hymen) being on the borders of Scotland must of course be amenable to the laws of that country: and since that country allows private marriages, it follows that those contracted in Gretna-Green are legally valid in every point of view.

ANSWERS

TO THE

QUESTIONS FOR YOUTH,

Proposed in the first Number of the second Volume.

(1) *By* Master T. WILSON.

Given the side of an equilateral triangle 36 chains. Required its area, and that of its circumscribing circle.

Solution by Master T. SMITH.

By Nesbit's Mensuration part II. p. 55, we have $.4330127 \times 36^2 = 561.1844592$ = area of the triangle. Also by p. 52 of the above work, we have $.5773503 \times 36 = 20.7846108$ = radius of the circumscribing circle, hence $20.7846108^2 \times 3.1416 = 1357.17134485$, are a of the circumscribing circle.

(2) *By* Master W. NICHOLSON.

Divide 100£. among A, B, and C, so, that A's share may be to B's as 3 to 4, and B's to C's as 6 to 5.

Solution by Masters A. WASHINGTON *and* T. MORTON.

First B (4) : A (3) :: B (6) : A ($4\frac{1}{2}$); then as A + B + C = $4\frac{1}{2}$ + 6 + 5 = $15\frac{1}{2}$, and as $15\frac{1}{2}$, : 100*l.* ::

$$\left.\begin{matrix} 4\frac{1}{2} : 29 & 0 & 7\frac{1}{2}\frac{30}{31} = \text{A's} \\ 6 : 38 & 14 & 2\frac{1}{4}\frac{0}{31} = \text{B's} \\ 5 : 32 & 5 & 1\frac{3}{4}\frac{25}{31} = \text{C's} \end{matrix}\right\} \text{share.}$$

Proof 100 0 0

Again by Masters C. THOMAS. H. ALLEN, *and* B. C. GOODISON.

Let x = B's share, then by the question A's share = $\frac{3x}{4}$, and C's share = $\frac{5x}{6}$; hence $x + \frac{3x}{4} + \frac{5x}{6} = 100$, whence $x = \frac{1200}{31}$, then A's share $\frac{900}{31} = £29\ 0\ 7\frac{1}{2}\frac{30}{31}$, B's share = £38 14 $2\frac{1}{4}\frac{0}{31}$, and C's share = £32 5 $1\frac{3}{4}\frac{25}{31}$.

(3) *By* Master B. C. GOODISON.

Given $x^2 - \frac{x}{y} = a$, and $y^2 + \frac{y}{x} = b$, to find x and y.

Solution by Master J. GREENHOUGH.

Here $\left(x^2 - \frac{x}{y}\right) \times \left(y^2 + \frac{y}{x}\right) = ab$, whence $x^2y^2 = ab + 1$, or $y = \sqrt{ab + 1} \div x$. Let this value of y be substituted in the first equation, and reduced; then we get $x = (a\sqrt{ab+1} \div \sqrt{ab + 1 - 1})^{\frac{1}{2}} = r$, and $y = \sqrt{(ab + 1)} \div r$.

Otherwise by Masters J. VINT *and* R. CLAPHAM.

$$x^2 - \frac{x}{y} = a \text{ or } x^2y - x = ay$$

$$y^2 + \frac{y}{x} = b \text{ or } y^2x + y = bx$$

$$x^3y^3 - xy = abxy \text{ by mult.}$$

$$x^2y^2 - 1 = ab \text{ by red.}$$

$$\text{then } xy = \sqrt{ab+1} \text{ and } y = \frac{\sqrt{ab+1}}{x}$$

By sub. in the first eq. $x^2 - \frac{x^2}{\sqrt{ab+1}} = a$, or $x^2 = \frac{a\sqrt{ab+1}}{\sqrt{ab+1}-1}$, therefore $x = \sqrt{\frac{a\sqrt{ab+1}}{\sqrt{ab+1}-1}}$. Let this $= c$, then $y = \frac{\sqrt{ab+1}}{c}$.

(4) Master W. GODWARD.

Required the side of a cube whose superficies is to its solidity as 6 to 11.

Solution by Masters J. WHITEHEAD, J. B. HAWKINS, *(Mr. Smith's Academy, York,) and* J. WHITAKER.

Let x = side of the cube required, then will $6x^2$ = the superficies, and x^3 = solidity, therefore per quest. as $6 : 11 :: 6x^2 : x^3$, whence $x = 11$, as required.

(5) *By* Master T. CUTTLE.

There is a conical frustum whose altitude is 8, and the diameters of its ends 4 and 6: divide it into two parts so that their surfaces may be equal, and find the solidities of those parts.

Solution by Master W. SHEPHERD.

Put x for the diameter of the section drawn parallel to the base; then $(8^2 + 1^2)^{\frac{1}{2}} = \sqrt{65} = s$ the slant height

of the given frustum; also $1 : s :: \frac{6-x}{2} : \frac{(6-x)s}{2}$ the slant height of the bottom part; and $1 : s :: \frac{x-4}{2} : \frac{(x-4)s}{2}$ the slant height of the top part. Now let $p = 3.1416$, and we have $\frac{(x^2 + 6^2)p}{4} + \frac{(6+x)p}{4} \times (6 - x)s =$ the whole superficies of the bottom part; and $\frac{(x^2+4^2)p}{4} + \frac{(x+4)p}{4} \times (x - 4)s =$ the whole superficies of the top part which by the quest. must be equal; whence $x = \left(26 + \frac{10}{s}\right)^{\frac{1}{2}} = 5.21922861$; again, $1 : 8 :: \frac{x-4}{2} : 4x - 16 = A =$ the alt. of the top part, and $1 : 8 :: \frac{6-x}{2} : 24 - 4x = a =$ the alt. of the bottom part; therefore, $(4^2 + 4 \times x + x^2)\frac{A}{3}.7854 = 81.8633938$ the solidity of the top; and $(6^2 + 6 \times x + x^2)\frac{a}{3}.7854 = 77.3110061$ the solidity of the bottom part. *Ans.*

(6) *By* Master J. H.

A person bought a quantity of cloth, and by selling one third of the same at 12s per yard, lost twenty per cent: how must he sell the remainder, so that he may lose nothing by his bargain?

Solution by Masters T. CUVELJE, T. W. ROBINSON, *and* W. GODWARD.

If 80 : 12*s.* :: 100 : 15*s.* prime cost per yard.

$\frac{12}{3}$ loss per yard.

If $\frac{2}{3} : \frac{3s.}{1} :: \frac{1}{3} :$ 1*s.* 6*d.* Hence 15*s.* + 1*s.* 6*d.* = 16*s.* 6*d.* *Ans.*

Again by Master J. WATSON,

It is evident that 20 per cent upon $\frac{1}{3}$ of the value of the cloth is equal to 10 per cent upon the other $\frac{2}{3}$, consequently as the price is 12*s* per yard the proportion will be £80 : 12*s*. :: 110*d*. : 16*s*. 6*d*. the price that the remainder must be sold at to cover the loss upon the other $\frac{2}{3}$.

(7) *By* JUVENIS.

Two sides of a triangle are 20 and 40, and the perpendicular from the vertical angle upon the third side, is 15: what is the area?

Solution by R. NECK.

First $\sqrt{40^2 - 15^2} + \sqrt{20^2 - 15^2} = 50.309748$, the base of the triangle. Hence $50.309748 \times \frac{15}{2} = 377.32311$ = the answer required.

Again by Master R. WEST.

First $\sqrt{40^2 - 15^2} = 37.08$ the greater segment, and $\sqrt{20^2 - 15^2} = 13.228$ = the less segment. Therefore $\frac{37.08 + 13.228 \times 15}{2} = 377.3$, the area required.

(8) *By* Master J. GREENHOUGH.

There are three numbers in harmonical proportion; the sum of the products of every two, is equal to the product of all three; the said products are in arithmetical progression, and the least is equal to the square-root of their sum: the arithmetical mean is equal to my age on 2nd of August 1815, and that age is required.

Solution by Masters A. WASHINGTON, T. MORTON, *and* T. SMITH.

Let x, y and z represent the three numbers, and by the nature of the question we shall have $xy + xz + yz = xyz$, $xy + yz = 2xy$ and $xy = \sqrt{xy + xz + yz}$, or $x^2y^2 = xy + xz + yz$. The three values of z, obtained

from these equations are $\frac{x^2y^2 - xy}{x+y}$, $\frac{xy}{xy-y-z}$, and $\frac{xy}{2x-y}$; equating the two last we obtain $y = 3$, and this value inserted in the first and second, and equated gives $\frac{9x^2 - 3x}{x+3} = \frac{3x}{2x-3}$, or $\frac{3x-1}{x+3} = \frac{1}{2x-3}$ which by reduction gives $x = 2$; hence $z = 6$, and $xz = 12$, the arithmetical mean, consequently the proposer was 12 years old, on the 2nd of August, 1815.

The following is a list of the answers to the questions for youth in Number I. Vol. II.

Masters Charles Thomas, *Eccleshill;* R. Neck, T. Morton, A. Washington, *Mathematical School, Reading;* J. Whitehead, *Mirfield;* J. Greenhough, *Bolton, near Bradford;* William Shepherd, *Eccleshill;* R. Chapman, *Mr. W. Sanderson's Academy, Aberford;* T. Smith, *Leeds;* and J. Watson, *Mr. Swale's Academy, Brunswick Place, Liverpool,* gave ingenious answers to all the questions.

Master J. Vint, *Idle,* answered 1, 2, 4, 5, 6, 7, 8.

Master J. Whitaker, *Mr. Leeming's Academy, Horsforth,* answered 1, 2, 4, 7.

Master H. Metcalf, *Royal Lancasterian School, Beverley,* answered 1, 2, 3, 4, 5.

Master R. West, *Haxey, near Gainsbro',* answered 1, 4, 5, 6, 7.

T. W. Robinson and T. Cuvelje, *Barningham Academy, near Greta Bridge* answered 1, 2, 4, 6, 7.

William Godward, *Free Writing School, Sheffield,* answered 1, 2, 4, 6, 7, and John Godward of the same school, and W. Kitson, *Windhill, near Bradford,* answered 1, 2, 7.

H. Allen and B. C. Goodison, *Rev. H. Davis' Seminary, Eaton Bishop, near Hereford,* answered 1, 2, 5, 7. J. B. Hawkins, *Mr. Smith's Academy, York,* answered 1, 4; and T. Clark, at the same Academy, answered the 7th.

ANSWERS

TO THE

MATHEMATICAL QUESTIONS

Proposed in the first Number of the second Volume.

(106) *By* JOB SHAINEN, *London.*

Determine two integer numbers such, that their difference, the difference of their squares, and the difference of their cubes may be rational squares.

Solution by Mr. S. JOHNSON, *Apperley Bridge.*

Put x and y for the numbers required, then since $x - y$, $(x-y) \times (x+y) = x^2 - y^2$ and $(x-y) \times (x^2 + xy + y^2) = x^3 - y^3$ are to be squares per question, all of which contain the square factor $x-y$, if $x-y$, $x+y$ and x^2+xy+y^2 be equated to squares, the question will be answered. Put $x = 5z$, $y = 3z$, then $x^2 + xy + y^2 = 49z^2$ is a square, and $x - y = 2z = \square = a^2$, or $z = \frac{a^2}{2}$, also $x + y = \square = 8z = 4a^2$; let $a = 2$, then $z = 2$, hence $x = 10, y = 6$ = the numbers required.

Otherwise by Messrs. T. CHARLTON and A. RICHARDSON.

This and indeed many other Diophantine problems may be expeditiously solved by the very useful and extensive tables of squares, cubes, and roots, in Vol. 1, of Dr. Hutton's Course of Mathematics, from which it readily appears that 10 and 6 will answer the conditions of the question; for $10 - 6 = 2^2$, $10^2 - 6^2 = 8^2$, and $10^3 - 6^3 = 28^2$.

Again by Mr. DICKENSON, *Hightown.*

Let $2x^2 - 2$, and $2x^2 + 2$ be the two numbers; then their difference, and the difference of their squares, will be both squares. And it only remains to make $48x^4 + 16 =$ the difference of their cubes, $=$ a square, or which is the same thing $3x^4 + 1$ is to be made a square. Assume $3x^4 + 1 = (2x^2 - 1)^2 = 4x^4 - 4x^2 + 1$. Hence $x = 2$, and the required numbers are 6 and 10.

Ingenious answers were also given by Messrs. Ambulator, Armitage, Baines, Blaymires, Cook, Crowther, Garside, Gaskin, Killingbeck, T. Morton, Page, Philomathes, Wadsworth, Washington and Whitehead.

(107) *By* Mr. J. BLAYMIRES, *Eccleshill.*

In three rectangular fields, the diagonals are 15, 17, and 25 chains respectively. The diameter of the greatest circle, that can be inscribed in each, between the diagonal and sides, is 6 chains. Required the contents of the fields.

Solution by Mess. JOHN GASKIN, *Penrith, and* JONATHAN CROWTHER, *Woodhouse-Grove.*

It is well known that in any right angled triangle the sum of the two sides including the right angle is equal to the sum of the hypothenuse and diameter of the inscribed circle, and the area $= \frac{1}{2}$ perim. $\times$ rad. of the same circle. Hence the three following results for the areas required.

	A.	R.	P.
$\overline{15+6+15} \times 3 = 108$ chains $=$	10	3	8
$\overline{17+6+17} \times 3 = 120$ ..... $=$	12	0	0
$\overline{25+6+25} \times 3 = 168$ ..... $=$	16	3	8
	39	2	16

Otherwise by AMBULATOR.

Put $x =$ the breadth of the first field; then $\sqrt{15^2 - x^2} =$ the length, and $15^2 - x^2 = \overline{21 - x}^2$ or $x = 12$, and

$\sqrt{15^2 - x^2} = 9$, consequently 108 sq. chains = the area. By proceeding in the same manner we obtain 120 and 168 sq. chains, for the contents of the other two.

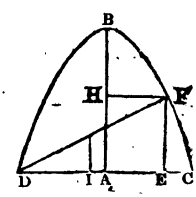
N.B. I have taken the radii of the circles 3 chains instead of "6 inches," as erroneously expressed on the cover of the last number.

True and ingenious solutions were also received from Messrs. F. and T. Charlton, Cook, Crowther, Dickenson, Moses Elbean, Esq. Garside, Gaskin, Holt, Johnson, Killingbeck, Page, Philomathes, Shenstone, Wadsworth, Washington, Watson, Whitehead, and Wiseman.

(108) *By* Mr. Fox, *Wakefield.*

Required the area of the greatest section that can be made by cutting a parabolic conoid with a plane passing through the extremity of its base; the diameter of the base being 6 and altitude 10.

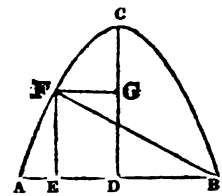
Solution by Mr. John Baines, Jun.



Let DBFC be the parabolic conoid, DF the transverse diameter of the elliptic section, and the other lines as in the figure. Put DE = $2x$, or DI = IE = x; then AE = FH = $2x - 3$, and $AC^2 : AB :: FH^2 : BH = \frac{40x^2 - 120x + 90}{9}$; but $AB - BH = AH = FE = \frac{120x - 40x^2}{9}$, and $FE^2 + DE^2 = FD^2 = \frac{1600x^4 - 9600x^3 + 14724x^2}{81}$ = the square of the transverse diameter; see prob. 10, part III. sec. VI. Hutton's Mensuration. Also $4x^2$ appears to be the square of the conjugate; hence $4x^2 \times \frac{1600x^4 - 9690x^3}{81} + 14724x^2$, or $x^2 \times (400x^4 - 2400x^3 + 3681x^2) = 400x^6 - 2400x^5 + 3681x^4$ must be a max. In fluxions, and reduced, &c. we get $x = 2{\cdot}5 - \sqrt{{\cdot}115} = 2{\cdot}160884$;

hence the transverse diameter is 9·14451, the conjugate 4.32177, and the area 31·03936.

Otherwise by Messrs. W. WATSON, *and* E. PAGE, *Beverley.*



Let ABC represent the paraboloid, and BF the transverse axe of the elliptic section. From the point F let fall a perp. FE upon the base AB, then will BE be the conjugate axe of the same section. Also, from the base bisected in D draw DC at right ∠s to AB, and FG ‖ to AB. Put $DC = a = 10$, $AD = DB = c = 3$, and $FG = ED = x$. Now by a property of the parabola $AD^2 : DC :: DE^2 = FG^2 : CG$, or as $c^2 : a :: x^2 : \frac{ax^2}{c^2} = CG \therefore EF = DG = a - \frac{ax^2}{c^2}$, and $BE = c + x$, now $\sqrt{BE^2 + EF^2} = BF = \sqrt{x^2 + 2cx + c^2 + a^2 - \frac{2a^2x^2}{c^2} + \frac{a^2x^4}{c^4}}$, then as the section is to be a max. the rectangle of the two axes will also be a max. that is $BF \times BE =$ a max. or $BF^2 \times BE^2 = x^2 + 2cx + c^2 \times x^2 + 2cx + c^2 + a^2 - \frac{2a^2x^2}{c^2} + \frac{a^2x^4}{c^4} = x^4 + 4cx^3 + 6c^2x^2 - a^2x^2 - \frac{a^2x^4}{c^2} + \frac{a^2x^6}{c^4} + 4c^3x + 2ca^2x - \frac{4a^2x^3}{c} + \frac{2a^2x^5}{c^3} + c^4 + c^2a^2 =$ a max. which when put into fluxions and reduced gives $3c^4x^3 + 12c^5x^2 + 12c^6x - 2a^2c^4x - 4a^2c^2x^5 + 6a^2x^5 + 4c^7 + 2c^5a^2 - 12a^2c^3x^2 + 10a^2cx^4 = 0$, or in numbers $150x^5 + 750x^4 - 819x^3 - 7371x^2 - 1863x = -14337$, hence $x = 1.3217$, $\therefore$ $BE = 4.3217$, and $DF = 9.1446$, and the area of the section $= 31.0392$.

Answers to this problem were also transmitted by Messrs. Cook, Crowther, Dickenson and Johnson.

(109) *By* Mr. W. RADCLIFFE.

A gentleman met a beggar to whom he gave $\frac{1}{n}$ of his money and s pence more, to the next which he met, he gave $\frac{1}{n}$ of what he then had, and $2s$ pence over; and proceeded after the same manner in relieving other objects; always giving s pence more, besides $\frac{1}{n}$ of his money left, until he had only p pence left, and then found that he had given an equal number of pence to each. How many did he relieve?

Solution by PHILOMATHES, *Idle*, AMBULATOR, *and* Mr. JOHN BLAYMIRES, *Eccleshill.*

Put $x =$ his money in pence, $y =$ the number of beggars, then $\frac{1}{n}x + s = \frac{1}{n}x - \frac{1}{n^2}x - \frac{1}{n}s + 2s$, and $\frac{x-p}{y} = \frac{1}{n}x + s$; from the former equation $x = \frac{n.ns-s}{}$, which written for its equal in the latter and reduced, there results $y = n - 1 - \frac{p}{ns}$ the number of beggars; where n, p, s may be taken any numbers at pleasure, provided $-1 - \frac{p}{ns}$ be a whole number, less than n.

Otherwise by Mr. J. CROWTHER *and* Mr. S. JOHNSON.

Let $x =$ the number of pence the gentleman had at the first; then he gave to the first beggar $\frac{x}{n} + s$, leaving in his pocket $\frac{x(n-1)}{n} - s$; to the second beggar $\frac{x(n-1)-ns}{n^2} + 2s$, leaving in his pocket $\frac{x(n-1)^2 - s(3n^2-n)}{n^2}$

to the third beggar $\frac{x(n-1)^2 - s(3n^2-n)}{n^3} + 3s$, and so on; but per question, he gave an equal sum to each beggar; and hence, by equating any two of the expressions for the sums given to the beggars, we find $x = s(n^2-n)$; therefore $\frac{x}{n} + s = \frac{s(n^2-n)}{n} + s = ns$, sum given to each beggar. Let n = the number of beggars, then $nns + p = (x =) s(n^2-n)$; hence $n = \frac{s(n^2-n)-p}{ns}$, number required.

Again by MOSES ELBEAK, *Plymouth.*

Let x = his money, then $\frac{x}{n} + s$ = sum given the first beggar, and $x - \frac{x}{n} - s$ = residue, consequently $\frac{x}{n} - \frac{x}{n^2} - \frac{s}{n} + 2s$ = sum given the second beggar, which per question is equal to the sum given the first; therefore $\frac{x}{n} + s = \frac{x}{n} - \frac{x}{n^2} - \frac{s}{n} + 2s$, or $\frac{x}{n^2} = s - \frac{s}{n}$, and $x = sn^2 - sn$, now $\frac{sn^2-sn}{n} + s = sn$ = each man's share, and $\frac{sn^2 - sn - p}{sn} = n - \frac{p}{ns} - 1$ = number of beggars.

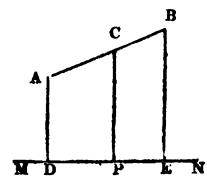
Similar answers were sent by Messrs. Armitage, Baines, F. Charlton, Cook, Dickenson, Hine, Holt, and Wiseman.

(110) *By* Mr. SETTLE, *Holbeck.*

Draw a straight line through a given point P so that the segments between P and perpendiculars let fall thereon from two given points, may have a given ratio.

Solution by Mr. CHARLES HOLT.

Let A, B and P be the given points; join AB, which divide at C in the given ratio; join C P, perpendicular to which thro' P draw MN, and let fall the perpendiculars AD and BE upon MN, and the thing is done. For by || lines DP : PE :: AC : CB, the given ratio.



Nearly thus was the question answered by Messrs. T. Charlton, Cook, Crowther, Dickenson, Garside, Johnson, Shenstone, and Whitehead.

(111) *By* Mr. W. WATSON, *Beverley.*

Given $2a^2\dot{x}x - ay\dot{x} + 3y\dot{y} = \dot{y} + ax\dot{y} - y\dot{y}$ to find the relation between x and y.

Solution by Mr. JOHN BAINES.

By transposition $2a^2\dot{x}x - \dot{y} + 4y\dot{y} = ax\dot{y} + ay\dot{x}$, and taking the fluents on both sides, $a^2x^2 - y + 2y^2 = axy$, $\therefore 2y^2 - y \cdot (ax + 1) = -a^2x^2$, or $y = \frac{ax + 1}{4} \pm \frac{1}{4}\sqrt{1 + 2ax - 7a^2x^2}$, the answer.

Otherwise by Messrs. WATSON *and* PAGE.

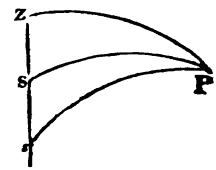
By transposition $2a^2\dot{x}x - ay\dot{x} - ax\dot{y} = \dot{y} - 4y\dot{y}$ its fluent $a^2x^2 - axy = y - 2y^2$, add $\frac{1}{4}y^2$ to each side of the equation, then $a^2x^2 - axy + \frac{1}{4}y^2 = y - 1\frac{3}{4}y^2$, its root, $\pm ax \mp \frac{1}{2}y = \sqrt{y - 1\frac{3}{4}y^2}$ or $\pm ax = \sqrt{y - 1\frac{3}{4}y^2} \pm \frac{1}{2}y \therefore \pm x = \frac{2\sqrt{y - 1\frac{3}{4}y^2} \pm y}{2a}$ the relation required.

Ingenious answers were also given by Messrs. A. Cook, Crowther, Garside, Johnson, and Wiseman.

(112) *By* Mr. J. BAINES, *Reading.*

In what latitude was the sun due east, one hour sooner on the 3rd of April 1815, than on the 3rd of June in the same year?

Solution by Messrs. CROWTHER, W. DICKENSON, *and* WILLIAM WISEMAN.



In the annexed figure let P be the pole, Z the zenith, Z*s* an arc of the prime vertical, *s* S the points in that arc where the sun crosses on each day respectively. Then in the spheric $\triangle$ SP*s*, we have given P*s* = 84°. 55′, the codeclination on April third, PS = 67° 45′ the codeclination on the third of June, and the included $\angle$ SP*s* = 15°; whence by case III oblique $\angle$'d spherics we find $\angle$ S*s*P = 38° 48′; and in the right $\angle$'d $\triangle$ PZ*s*, as rad : co-dec. *s*P :: sin. $\angle$ Z*s*P : sin ZP = cos. lat. 51° 23′ the latitude required.

The same otherwise by Mr. COOK.

Put a = cos. 15°, T and t = tang. of sun's declination on April third and June third respectively, and x = cot. latitude. Then from the principles of spherics, we have, rad. 1 : $\left[\begin{matrix}\mathrm{T}\\ t\end{matrix}\right]$ :: x : $\left[\begin{matrix}\mathrm{T}x\\ tx\end{matrix}\right]$ = cosines of the hour $\angle$s from noon when the sun is on the prime vertical on $\left[\begin{matrix}\text{April } 3\\ \text{June } 3\end{matrix}\right]$. Then by trig. the cos. of the diff. of these $\angle$s is = $\mathrm{T}tx^2 + \sqrt{(1 - (\mathrm{T}^2 + t^2)\, x^2 + \mathrm{T}^2t^2x^4)}$ = a by the question, or by transp. and squaring, $1 - (\mathrm{T}^2 + t^2) \cdot x^2 + \mathrm{T}^2\, t^2x^4 = a^2 - 2a\mathrm{T}tx^2 + \mathrm{T}^2t^2x^4$; hence

$$x = \sqrt{\frac{1 - a^2}{\mathrm{T}^2 + t^2 - 2a\mathrm{T}t}} = .798818 = \text{co-tang. } 51°\ 22'\ 54''$$

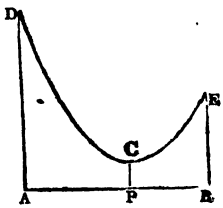
the latitude required.

Good solutions to this question were received from Messrs. Crowther, Garside, Gaskin, Hine, Johnson, Page and Watson.

(113) *By* JOB SHAFNEN, *London.*

From any point P in the given right line AB = 10 inches, let a perpendicular PC = $(AP^2 + PB^2) \div (2AP + PB)$ be erected, it is required to determine the area of the curve, which is the locus of C.

Solution by Mr. S. JOHNSON.



Let AB = 10 = a, AP = x, PC = y, then per quest. $y = \frac{x^2 + (a - x)^2}{x + a}$ $= 2x - 4a + \frac{5a^2}{a+x}$ and $\dot{y}x$ $= 2x\dot{x} - 4a\dot{x} + \frac{5a^2\dot{x}}{a+x}$, the fluent of which is $x^2 - 4ax + 5a^2 \times$ h. l. $(a + x)$. Now when $x = 0$ the area $= 0$, hence the fluent corrected is $x^2 - 4ax + 5a^2 \times$ h. l. $\frac{a+x}{a}$, which when $x = a$ becomes $5a^2 \times$ h. l. $2 - 3a^2$ $= 500 \times .6931472 - 300 = 46.5736 =$ area required.

The same by Mr. WISEMAN, *Hull.*

Put AP = x, PC = y, AB = $10 = a$, and let the area APCD be denoted by A. Then PB being $= a - x$, we have $y = \frac{x^2 + a^2 - 2ax + x^2}{2x + a - x} = 2x - 4a + \frac{5a^2}{x+a}$; whence $\dot{A} = y\dot{x} = 2x\dot{x} - 4a\dot{x} + \frac{5a^2\dot{x}}{a+x}$, and therefore $A = x^2 - 4ax + 5a^2 \times$ h. l. $\overline{a + x} + C$. But when $A = o$, $x = o$, therefore $C = -5a^2 \times$ h. l. a, whence $A = x^2 - 4ax + 5a^2 \times$ h. l. $\overline{a + x} - 5a^2 \times$ h. l. a; or $A = x^2 - 4ax + 5a^2 \times$ h. l. $\frac{a+x}{a}$, which (when $x = a$) is $= -3a^2 + 5a^2 \times$ h. l. $2 = a^2 \times (5$ h. l. $2 - 3) = a^2 \times .465735 = 46.5735$, the area required.

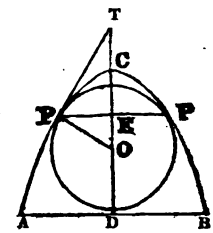
Answers equally true and ingenious were received from Messrs. Baines and Crowther.

(114) *By* Mr. S. TREEBY, *Plymouth.*

Determine the radius OP of a circle touching a given parabola APCP'B in P, P', when the circle and parabola have a common tangent PT.

Solution by Messrs. COOK, *and* CROWTHER.

Draw the axis CD, and PP', cutting CD in E. Then in number 4 of this work it is demonstrated that ED = AD; also OE = $\frac{1}{2}$ the parameter of the parabola, i.e. = $\frac{\frac{1}{2}AD_2}{CD}$, and consequently OD = OP = AD $-\frac{1}{2}p = AD - \frac{\frac{1}{2}AD^2}{CD}$ the radius of the inscribed circle.



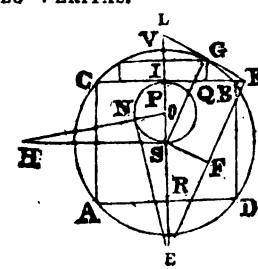
Messrs. Baines, Dickenson, and Johnson sent very ingenious solutions to this question.

(115) *By* MECHANICUS.

A Person has a circular table whose diameter is 4 feet: out of this table he intends to make three rectangular ones thus,—after having made one, he will join the opposite segments cut off, out of which he will make other two; what will be the dimensions of those tables, that, when taken together, they may have the greatest quantity of wood in them?

Solution by PHILO VERITAS.

The greatest rectangle that can be inscribed in the given circle is the square ADBC, as is too well known to require a demonstration here: consequently the four remaining segments will be equal to each other. To the point G draw the tangent LK meeting CB



produced in K and RL, drawn through the centre of the circle perpendicular to CB in L. Then by theo. 8 Simpson on the max. et min. the rectangle inscribed in the segment CGB is a maximum when GL = GK. Draw KE ‖ to SG joined, or perpendicular to LK, meeting RL produced in E, and on EK demit the perp. SF. Because GL = GK, by parallels, SE = SL, and FE = FK = SG; therefore EK is = the diameter and FE or FK is = the radius of the given circle. Now since EFS and EPK are right angles, a circle will pass through the points F, S, P, K: therefore ES.EP = EF.EK = $2SG^2$ = square ADBC. Hence this

CONSTRUCTION. On SP as a diameter describe the circle SNPS, and having drawn SH perpendicular to SP and = AD, draw HO to its centre O, cutting the circumference in N; erect NE perpendicular to HO meeting SP produced in E; take SL = SE, and from L draw the tangent LG: on PL, CB demit the perpendiculars GI, GQ: so shall GI = ½ the length, and GQ = ½ the breadth, of the required rectangles. (Vide Simpson's Geom. 18. 5.)

COMPUTATION. Since the diam. = 4 = EK, the area of the square ADBC = 8, SP = $\sqrt{2}$, and OS = $\frac{1}{2}\sqrt{2}$ = OP: hence OH = OE = $\sqrt{OS^2 + SH^2} = \sqrt{\frac{1}{2} + 8}$ = $\frac{1}{2}\sqrt{34}$; then EP = OE + OP = $\frac{1}{2}\sqrt{34} + \frac{1}{2}\sqrt{2}$, and RE = PL = 2 GQ = OE − 3OS = $\frac{1}{2}\sqrt{34} - \frac{3}{2}\sqrt{2}$ = .794155 feet = breadth, also PK = 2GI = $\sqrt{EK^2 - EP^2} = \sqrt{7 - \sqrt{17}}$ = 1.696141 feet = length of the rectangles.

Otherwise by Mr. JOSHUA GARSIDE, *Mirfield, and* Messrs. W. WATSON *and* E. PAGE, *Beverley.*

Put $a = 4$, and x = SP half the length of the greatest table, then CP = $\sqrt{4 - x^2}$ = half the breadth; whence $x \times \sqrt{4 - x^2} = \frac{1}{4}$ of the area, a maximum: its fluxion put = o, and reduced gives $2x$ = 2.8284272 = side of a square inscribed in the given circle. Again, put b = SP = $\sqrt{2}$, and y = GI = half the length of

the less tables, then SP $= \sqrt{4 - y^2}$, and PI $=$ GQ $= \sqrt{4 - y^2} - b =$ half the breadth; therefore $\frac{1}{4}$ of the area $= y\sqrt{4 - y^2} - by$ a max. which put into fluxions, and reduced, we get $y^4 - \frac{1}{2}y^2 = -2$: whence $2y = 1.696141$ feet, the length and the breadth $=$ .7941556 feet the dimensions required.

Another solution by Mr. H. ARMITAGE.

By theo. 9th Simpson on the max. et min. the greatest rectangle will be a square whose side $= 2.828427$ which put $= 2b$, and the height of the seg. $= .5857865$ put $= a$, $2x = 2GI =$ length of the rectangle, and $y =$ its breadth: then by schol. to theo. 8th Sim. on the max. et min. the greatest parallelogram that can be described in any curve concave to its axis, will be when the subtangent is $=$ to the base of the parallelogram then $2x - b =$ BK, $2x + b =$ CK, $(2x - b) \times (2x + b) - x^2 =$ GQ$^2 = y^2$, and $x^2 = (2b + a + y) \times (a - y)$, by reduct. $y = -\frac{3b}{4} \pm \frac{\sqrt{24ab + 12a^2 + 5b^2}}{16} =$ .397078 and $2x = 1.6961408$.

Ingenious solutions were received from Messrs. Ambulator, Dickenson, Holt, Johnson, and Shenstone.

(116) *By* Mr. DARBY.

Required a solution independent on algebra, to a question in Dodson's Repository, page 62, vol. 1, which is as follows: If A and B together can perform a piece of work in 8 days; A and C, in 9 days, and B and C in 10 days: how many days will it take each person alone to perform the same work?

Solution from Dodson's Repository, communicated by ANASTHIMENES.

Here 8, 9 and 10 days become $\frac{1}{8}$, $\frac{1}{9}$ and $\frac{1}{10}$, reduced to a common denominator become $\frac{90}{720}$, $\frac{80}{720}$ and $\frac{72}{720}$. Therefore suppose the work consist of 720 parts and

90, 80, and 72 parts will be the work done in a day by AB, AC, and BC = 242 parts ÷ 2 = 121 the work done by all in a day.

$$\text{As } 121 \text{ Pts.} \left\{\begin{matrix} 72 = 49 \\ 80 = 41 \\ 90 = 31 \end{matrix}\right\} : 1 \text{ Day.} :: 720 \text{ Pts.} : \left\{\begin{matrix} 14\frac{4}{49} \text{ Days. by A alone.} \\ 17\frac{7}{41} \text{ ... B .. ...} \\ 23\frac{4}{31} \text{ ... C .....} \end{matrix}\right.$$

Otherwise by Mr. H. ARMITAGE, *Rochdale.*

First find what will be done by them conjointly in 360 days, viz. as 8 : 1 :: 360 : 45 work done by AB in 360 days. As 9 : 1 :: 360 : 40 work done by AC in the same time. As 10 : 1 :: 360 : 36 work done by BC. Then 45 — 40 = 5 = difference of B's and C's work in 360 days, 36 = sum of their works: hence B's work = $20\frac{1}{2}$, C's = $15\frac{1}{2}$, and A's = 45 — $20\frac{1}{2}$ = $24\frac{1}{2}$; then 360 divided by each of their work quotes $17\frac{7}{41}$ B's, $23\frac{4}{31}$ C's, and $14\frac{4}{49}$ A's.

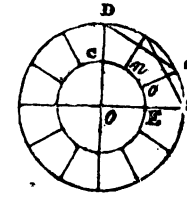
Answers to this question were also transmitted by Messrs. Ambulator, Baines, T. Charlton, Crowther, Garside, Gaskin, Holt, Johnson, Page, Pashley, Washington, Watson, and Whitehead.

(117) *By* Mr. JOSHUA GARSIDE, *Mirfield.*

A circular piece of waste ground, quite level, and 50 chains in diameter is to be divided Geometrically, amongst 16 proprietors, so that all the parts shall be of the same area, and of the same perimeter, and all this to be done, not by drawing radii, or the other lines from the center to the circumference, nor by the method invented by Dr. Hutton, as published in his tracts. Now the repective areas and perimeters of the required parts may be hence determined—query how?

Solution by Messrs. CROWTHER *and* JOHNSON.

CONSTRUCTION. Draw the radius OD, bisect it in C, with centre O and rad. OC describe another circle, and through both circles draw 2 diameters intersecting at right angles. In any of the quadrants ODS thus described apply the chords Dr,



Sm, equal to the radius; then we know the arc DS will be trisected by the points m, n. In like manner let the lesser quadrant CE be trisected in n and o, and join nm, or: and so on for the other quadrants. Then in the first place the radius of the inner circle being $\frac{1}{3}$ that of the outer one, its area will be $\frac{1}{9}$ of the greater circle, and consequently each of the divisions formed by the intersecting diameters equal $\frac{1}{18}$ of the whole area, or $\frac{1}{4}$ of the quadrant ODS; the remaining divisions in the outer are also evidently similar to one another, and each equal to a quadrant of the lesser circle. It therefore only remains to prove that the perimeters are equal. Now OD is by construction $= 2$ OC: therefore OC + OE + CE $= 2$CD + Cn + (nE $=$) Dm $=$ CD + nm + Cn + Dm. Q.E.D.

This question was also answered by Messrs. Garside, Page, and Watson.

(118) *By* I. R. G.

A and B, having each a string of a different length, would measure a triangular field: A found the length of the first side to be 5 strings, and that of the second, 6 strings 4 yards; B found the same sides to be 6 strings 2 yards, and 8 strings, respectively; they then find the third side $=$ 4 times the length of their strings when connected together: whence the area of the field is required.

Solution by Mr. J. Garside.

First put the length of A's string $= x$, and that of B's $= y$, then by the question $5x = 6y + 2$, and $6x + 4 = 8y$. Now from the first equation $x = \frac{6y+2}{5}$, and from the latter $x = \frac{4y-2}{3}$, consequently $\frac{6y+2}{5} = \frac{4y-2}{3}$, hence by reduction $y = 8$ yards, and from a former equation $x = 10$ yards. Now the lengths of the string being found, the lengths of the sides of the field are also found, namely 50, 64, and 72 yards respectively, whence the area $= 1560.573932$ yards.

Otherwise by Mr. J. KILLINGBECK.

Put x = the length of A's string, and y = that of B's. Then the length of A's first side $= 5x$, and that of the second $= 6x + 4$, and the same sides measured by B's string are $6y + 2$, and $8y$. By equality we have $6y + 2 = 5x$, and $6x + 4 = 8y$, from the latter eq. $y = \frac{6x + 4}{8}$ this value of y substituted in the first eq. x is found $= 10$, and $y = 8$, $\therefore$ the sides are 72, 64, and 50 yards. Hence the area of the $\triangle = 1560.5739$ square yards.

Again by Mr. J. SHENSTONE.

Let x = A's string, y = B's; then per question $5x = 6y + 2$. And $6x + 4 = 8y$, $\therefore x = \frac{6y+2}{5} = \frac{8y-4}{6}$.

Hence x will be found equal 10, and $y = 8$ yards. Also the sides of the fields $= 50$, 64, and 72 yards respectively. Hence the area $= 1560.573$ yards.

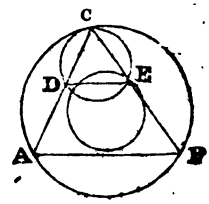
Ingenious solutions were also received from Messrs. Armitage, J. Baines, T. Charlton, Cook, Crowther, Dickinson, Gaskin, I. R. G., Holt, Johnson, Page, Philomathes, Richardson, Washington, Watson, and Whitehead.

(119) *By* GEOMETRICUS, *Rotherham.*

To inscribe a triangle in the greater of two given concentric circles, so that two of its sides may pass through two given points, and the remaining side touch the less given circle.

Solution by Mr. J. CROWTHER, *Woodhouse Grove.*

CONSTRUCTION. Let D, E be the 2 points; join DE, and upon DE describe a segment of a circle to contain an $\angle$ = that contained by the segment cut off by the chord which touches the lesser circle, and from C either of the points where the segment cuts the



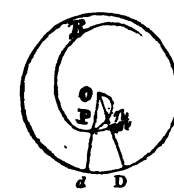
greater circle, through D and E respectively draw CA, CB; join AB, and ABC is the triangle required. For the $\angle$ ACB, or DCE is constant and is by construction = the $\angle$ cut off by a chord touching the inner circle; consequently AB touches that circle. If the arc of the segment DCE do not intersect or touch the outer circle the question is impossible, and in the latter case the $\angle$ ACB or the chord AB is a maximum.

Similar solutions to this question were received from Messrs. Cook, Holt, Johnson, Page, and Watson.

(120) *By* Mr. Wm. Wright, *Bonby, near Barton upon Humber.*

O is the center, OD the radius of the circle revolving about O with a given uniform velocity. Now, suppose a point P begin to move from O along OD with a motion so regulated that the curve generated by the compound motion shall be described with a given velocity; I would know what will be the length of the said curve.

Solution by Mr. J. Crowther.



Let O*n*PR represent the curve described by P; draw the radii OD, O*d* indefinitely near each other and intersecting the curve in *n* and P; also draw *m*P perpendicular to OD. Let OP = x, dD = $\dot{y}$, OD = r; then by Sim. sectors, as $r : \dot{y} :: x :$ $\frac{x\dot{y}}{r}$ = Pm; but Pn^2 = mn^2 — Pm^2, i.e. $\dot{x}^2 = \dot{z} - \frac{x^2\dot{y}^2}{r^2}$

Let $a : b$ be the ratio of the velocities of D and P, then $a : b :: \dot{y} : \dot{z}$, and $\dot{y} = \frac{a\dot{z}}{b}$, which value being substituted for $\dot{y}$ in the preceding expression we have $\dot{z}^2$ —

$\frac{a^2x^2\dot{z}^2}{b^2r^2} = \dot{x}^2$; or $\dot{z} = \frac{br\dot{x}}{\sqrt{b^2r^2 - a^2x}}$. The fluent of this equation gives us z = circular arc to radius $\frac{br}{a}$ and sine x. Hence, if the velocities be equal, the points P and D will both have passed over a space equal to the arc of a quadrant, and will meet at the end of it; (for, by substituting in the last equation On instead of $\dot{z}$, its value in terms of $\dot{x}$ and $\dot{y}$ and taking the fluent we have $y = \frac{a}{b} \times$ circ. arc to rad. $\frac{br}{a}$ and sine x, and consequently when $x = OD$, y and of course the curve described by P are both = a quadrant.)

Otherwise by Mr. A. Cook.

While the radius OD moves into any position OnPR, suppose the point P to have moved from O to P and traced out the curve OnP. Draw Ond indefinitely near OPD; also draw Pm perp. to Od. Let the veloc. of D : veloc. P :: m : n, put $OD = r$, arc $DD = x$, $OP = y$, then the curve $On\text{P} = \frac{nx}{m}$, $Dd = \dot{x}$, $mn = \dot{y}$, $Pn = \frac{n\dot{x}}{m}$; and by sim. tri. $Pm = \frac{y\dot{x}}{r}$. Now $Pm^2 + mn^2 = Pn^2$; that is $\frac{y^2\dot{x}^2}{r^2} + \dot{y}^2 = \frac{n^2\dot{x}^2}{m^2}$, hence $\dot{x}^2 = \frac{\dot{y}}{\frac{n^2}{m^2} - \frac{y^2}{r^2}}$ or $\dot{x} = r \times \frac{\dot{y}}{\sqrt{\frac{n^2r^2}{m^2} - y^2}}$: the fluents are x = circ. arc to rad. r and sine $\frac{my}{nr}$; hence the curve $On\text{P} = \frac{nx}{m} = \frac{n}{m} \times$ circ. arc to rad. r, and sine $\frac{my}{nr}$.

An ingenious solution was received from Mr. Johnson.

(121) *By* FACILIOR.

At the same time that a ball was let fall from the top of a tower, another was projected upward with a velocity of 100 feet per second, and it was observed that when the former arrived at the bottom, the latter was descending and at the distance of 20 feet: required the height of the tower.

Solution by Mr. A. COOK.

The projectile velocity upwards being 100 feet per second, we shall have by the laws of gravity, $\frac{100^2}{4 \times 16\frac{1}{12}}$ = 155.44 feet = the height ascended to by the ball before it stops, and begins to fall downward; therefore the time of the ball's arriving again within 20 feet of the top of the tower is $= \frac{100}{2 \times 16\frac{1}{12}} + \sqrt{\frac{135.44}{16\frac{1}{12}}} =$ 3.1088 + 2.9019 = 6.0107 seconds; this by the question is also equal to the time occupied by a body in falling by the force of gravity through the height of the tower, hence $16\frac{1}{12} \times 6.0107^2 = 581.0669$ feet the height required.

Otherwise by Mr. W. DICKINSON.

Put 20 feet = a, 100 = v, and $g = 16\frac{1}{12}$, then $\frac{v^2}{4g}$ = height the ball will ascend, $\frac{v}{2g}$ = time of ascension, $\frac{v^2}{4g} - a$ = space fallen after it begins to descend till it arrives within 20 feet of the bottom, and $\sqrt{\left(\frac{v^2}{4g} - a\right) \div g}$ $= \sqrt{\frac{v^2 - 4ag}{4g^2}}$ = time of descending that space, then

$\frac{v}{2g} + \sqrt{\frac{v^2 - 4ag}{4g^2}}$ = time the ball will be descending from the top of the tower. Hence $g\left(\frac{v}{2g} + \sqrt{\frac{v^2-4ag}{4g^2}}\right)^2$ = 581.0669 feet = height of the tower.

Also by Mr. J. GARSIDE.

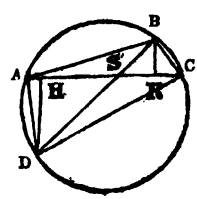
Put $g = 16\frac{1}{12}$ feet, the space fallen through in the first second of time, $v = 100$, the given velocity, t = the time of the ball's ascending. Then by Dr. Hutton's course vol. 2. p. 149. $t = \frac{v}{2g} = 3\frac{61}{194}''$ the time of the ball's ascending. Again $\frac{1}{2}tv = 155\frac{85}{193}$ feet = the height ascended by the ball. Now $155\frac{85}{193} - 20 = 135\frac{85}{193} = s$ feet, and $\sqrt{\frac{s}{g}} = 2\frac{9}{10}''$ = the time of descending through s feet, therefore $3\frac{61}{193}'' + 2\frac{9}{10}'' = 6\frac{17}{1930}'' = t$, the time of the ball's descending from the top to the bottom of the tower, whence the height of the tower is readily had which $= gt^2 = 580.701$ feet, the answer.

Solutions were also communicated by Messrs, Baines, T. Charlton,, Crowther, Gaskin, Hine, Holt, Johnson, Page, Pashley, Philomathes, Richardson, and Wadsworth.

(122) *By* Mr. S. JOHNSON, *Apperley-bridge.*

Find general expressions for the area, sides, Diagonals, and the diameter of the circle circumscribing a trapezium, the sum of its diagonals being a square number.

Solution by the Proposer.



Suppose ABCD the required trapezium, S the intersection of the diagonals AC, DB, and DH, BR perp. to AC. Put AB $= (m^2 + n^2)\ 2fg$, AR $= (m^2 - n^2)\ 2fg$, BC $= (f^2 + g^2)\ 2nm$, CR $= (f^2 - g^2)\ 2nm$, BS $= (n^2 + g^2)\ 2fm$, SR $= (n^2 - g^2)\ 2fm$; then by the property of the circle DS =

$\frac{AS \times SC}{BS} = \frac{(m^2-n^2)fg-(n^2-g^2)fm \times (f^2g^2)2nn+(n^2-g^2)2fm}{(n^2+g^2)fm}$
$= (mg - n^2) \times (m + g) \times (nf - g^2) \times (f + n) \times 2 \div (n^2 + g^2)$, also $DC = \frac{AB \times DS}{AS} = \frac{(m^2 + n^2)fg}{(n^2 + g^2)fm} \times$
$\overline{(f^2 - g^2)\,2nm + (n^2 - g^2)\,2fm} = 2g\ (m^2 + n^2) \times (nf$
$- g^2) \times (f + n) \div (n^2 + g^2)$ and $AD = \frac{BC + DS}{SC}$
$= \frac{(f^2 + g^2)\,nm}{(n^2 + g^2)\,fm} \times (m^2 - n^2)\,2fg - (n^2 - g^2)\,2fm =$
$2\,(f^2 + g^2) \times n\,(mg - n^2) \times (m + g) \div (n^2 + g^2)$; also the diameter of the circumscribing circle $= (m^2 + n^2) \times (f^2 + g^2)$. Now if AD, DS and DC are to be integer numbers, one or more of the factors in the expressions for their equals must be assumed $=$ to $n^2 + g^2$; but as the sum of the expressions for AC and BD is to be a square, let n be taken $= g$; in which case the diagonals intersect at right angles; and $DS = (m - n) \times (m + n) \times (f - n) \times (f + n)$; $BS = 4n^2fm$, $AR = (m^2 - n^2)\,2fn + (f^2 - n^2)\,2mn$, $CR = (f^2 - n^2)\,2mn$; hence $BD = (m^2 - n^2) \times (f^2 - n^2) + 4n^2fm$; $AC = (m^2 - n^2)\,2fn + (f^2 - n^2)\,2mn$; therefore $AC + BD = (m^2 - n^2) \times (f^2 - n) + 4n^2fm + (m^2 - n^2)\,2fn + (f^2 - n^2)\,2mn = (m^2 + 2mn - n^2) \times (f^2 + 2fn - n^2) = \square$; let each of these factors be equated $=$ a square. But $(f^2 + 2fn - n^2) = (f + n)^2 - 2n^2 = \square$, assume its root $= f + n - \frac{n}{q}$; hence $(f + n)^2 - 2n^2 = \overline{f + n - \frac{n}{q}}^2 = (f + n)^2 - 2\frac{n}{q}(f + n) + \frac{n^2}{q^2}$ from which $n = 2q, f = 2q^2 - 2q + 1$; hence by substituting $2q$ for n in the expression $m^2 + 2mn - n^2$ it becomes $m^2 + 4mq - 4q^2 = m^2 + 4q\,(m - q) = \square$: make the root of this square $= r\,(m - q) - m$; then $m^2 + 4q\,(m - q) = r^2\,(m - q)^2 - 2rm\,(m - q) + m^2$

from which $q = r^2 - 2r$, $m = r^2 + 4$; hence it is evident that r may be taken at pleasure provided it be greater than 20; let $r = 4$, then $m = 20$, $q = 8$, $n = 16$, $f = 113$, from which the sides &c. of the trapezium are found, and if $2r^4 - 8r^3 + 6r^2 + 4r + 1$, $2r^2 - 4r$, $r^2 + 4$ be substituted for f, n, and m, there will be obtained the general expressions required.

This is the only true solution we have received to this difficult question.

Question 5229 *Gentleman's Diary*, 1785.

Given $\frac{\ddot{y}y}{\dot{z}^2} + n^2y^2 + p^2 \times \text{: cos. } {}^2z = 0$, to find y without series, n and p being any constant quantities.

Answered by EUMENES.

In the given equation put N for the number whose hyp. log. $= 1$, then $z\sqrt{-1}$ = hyp. log. of $Nz\sqrt{-1}$ = hyp. log. of M^z the square of the cosine of $z = \frac{1}{4}M^{2z} + \frac{1}{2} + \frac{1}{4}M^{-2z}$; substitute this for cos. ^2z and assume $y = AM^z + BM^{-z}$, then $y^2 = A^2 \cdot M^{2z} + 2AB + B^2 \cdot M^{-2z} \frac{\dot{y}^2}{\dot{z}^2} = AM^z - BM^{-z}, \frac{\ddot{y}}{\dot{z}^2} = AM^z + BM^{-z} = y, \frac{\ddot{y}y}{\dot{z}^2} = y^2$, and $(1 + n^2) \cdot A^2M^{2z} + 2AB + B^2 \cdot M^{-2z} + \frac{1}{4}p^2 \times (M^{2z} + 2 + M^{-2}) = 0$, hence $(1 + n^2)\,A^2 = -\frac{1}{4}p^2$, $(1 + n^2)\,B^2 = -\frac{1}{4}p^2$, consequently $A = B = \frac{1}{2}\frac{p}{\sqrt{1 + n^2}}\sqrt{-1}$, also $2AB = 2A^2$, and $2A^2 \times (1 + n^2) = -\frac{1}{2}p^2$. $A = \frac{1}{2}\cdot\frac{p}{\sqrt{(1 + n^2)}}\sqrt{-1}$, that is both the sets of homologous terms give the same value of A and B. If they had not, it would have been necessary that p and n should have a particular relation, which would hence have been deter-

mined, so that they could not then have been any constant quantities whatever as the question requires. Substituting $p^2 = -q^2$ and the values of A and B in the assumed one for y, we find $y = \frac{q}{2\sqrt{(1+n^2)}} \times (M^z + M^{-z}) = \frac{qx}{\sqrt{(1+n^2)}}$ where x = cos. of the circ. arc. z to rad. unity. If the expression $\frac{\ddot{y}y}{\dot{z}^2} + n^2y^2 + p^2 \times$: cos. $2z = q^2$, by proceeding in the same manner we obtain $y = \frac{qx\sqrt{2}}{\sqrt{(1+n^2)}}$, or if the said expression $= 0$, and we take the fluxion thereof, and thence find y, the very same value will come out; but it is not from hence to be concluded that this will answer in all cases, be the value of the expression nothing or what it will, for it will only answer in the one particular case, when it is $= -p^2$. Hence it appears that the direction given by *fluxionary writers*, to take the fluxion of the given expression, and to find the value of the unknown quantity as the question then stands, will mostly in the higher orders of fluxions lead us into errors. Because the expression thrown into fluxions gives the value of y above, which, substituted in the expression made equal to nothing is found not to answer, it is not thence to be concluded that therein y has no proper value, but only that it has none of that form, or attainable in that manner. And hence occasion may be taken to remark on the hopelessness of the undertaking of *M. M. Fontaine, Condorcet, &c.* to find a general formula applicable in all cases; indeed, the forms already given by them and others are for the most part much too general, so that it is far more troublesome to apply them to any particular case that may occur, than it would be to resolve it without them.

Here follows the demonstration belonging to Mr. CROWTHER'S solution of the 105th question, for which there was not room in our last number, page 72.

DEMONSTRATION. Draw OP, OB, and let nP be produced to meet BH parallel to OP in H. Suppose any other line bc to be drawn meeting the circle O in b, and P in c: join O, b, c, P; let Ph parallel to Ob meet bh parallel to BH: and join c, h. Then by construction and parallel lines nC = OB = PH; therefore CH = Pn; also BH (by cons.) = OP; hence the $\triangle$s OnP, BCH are similar and equal. Again, for the like reasons bh = OP = BH, Ph = Ob = PH, and PC (PC) + Ph = CH; also $\angle cbh = \angle$ CBH. Let then the $\triangle$ bch be supposed to be placed upon the $\triangle$ BCH so that bh may coincide with BH, then will bc coincide with BC; but ch (Simpson's Geom. I. 19.) is less than (cP + Ph) CH; hence, if Hm be drawn perpendicular to BD, ch will lie between C and m (Ibid. I. 20); bc is then greater than BC, which is consequently a min. Q.E.D. Hence also BE + CF, or AB + CD is a maximum, and AD is parallel to the given line by construction.

SCHOLIUM. The line OP, joining the two centres is divided by AD into 2 parts having the same ratio as the 2 radii, for by Sim. tri. as OB : PC :: OG : PG; hence when the circles are equal, AD will bisect OP, and the 2 intercepted chords will be equal.

Two Cambridge problems with their solutions by

AMICUS.

1. Two roots of the equation $x^4 - x^3 - 11x^2 + 9x + 18 = 0$, are of the form $+ a$, $- a$. Find all the roots.

Solution.

Let $+ a$, $- a$, b and c denote the roots required: then $(x + a) \times (x - a) \times (x - b) \times (x - c) = x^4 - x^3 - 11x^2 + 9x + 18 = x^4 - bx^3 - cx^3 + (bc - a^2)\, x^2 + (ba^2 + ca^2)\, x - bca^2$ Equating the co-efficients of the homologous terms, we have $b + c = 1$, bc

$-a^2 = -11$, $(b+c)\,a^2 = 9, = a^2 \therefore a = \pm 3$, $bc = -18 + a^2, = -2$: from this and the equation $b + c = 1$, we find $b = 2$, and $c = -1$: so that the four roots are $+3$, -3, $+2$ and -1.

2. Find the equation to the curve, the length of whose tangent between any point and the axis is a constant quantity.

Solution.

Let $x =$ the abscissa and $y =$ the ordinate of the required curve, then the subtangent $= \frac{yx}{\dot{y}}$, and putting $a =$ the constant quantity, we have by Eu. 47, 1, $\frac{y^2\dot{x}^2}{\dot{y}^2} + y^2 = a^2$. Assume $z\dot{y} = \dot{x}$, then by substitution $y^2z^2 + y^2 = a^2$: hence $y = \frac{a}{\sqrt{1+z^2}} = \frac{a}{v}$, putting $v = \sqrt{1+z^2}$; $\dot{y} = -\frac{a\dot{v}}{v^2}$, $\dot{x} = \dot{y}z = -\frac{a\dot{v}\sqrt{v^2-1}}{v^2}$ $= \frac{a\dot{v} - av^2\dot{v}}{v^2\sqrt{v^2-1}}$, and taking the fluents $x = \frac{a}{v}\sqrt{v^2-1}$ $- a \times$ hyp. log. $\overline{v + \sqrt{v^2-1}} = \sqrt{a^2 - y^2} - a \times$ hyp. log. $\frac{a + \sqrt{a^2-y^2}}{y}$, the equation of the curve required; from which it appears that the curve is concave to its axis, having four infinite arcs, and each two of them on the same side of the axis forming a *cuspes* at the distance of the constant quantity a therefrom.

THE

LEEDS CORRESPONDENT,

A

Literary, Mathematical and Philosophical Miscellany.

Tarda fluunt ingrataque tempora, quæ spem
Consiliumque morantur agendi gnaviter id, quod
Æquè pauperibus prodest, locupletibus æquè;
Æquè neglectum pueris senibusque nocebit.

HORAT.

No. III. JANUARY, 1817. Vol. II.

Translations are respectfully requested of those, among our junior correspondents, who have not attained to sixteen years of age.

JUVENILE DEPARTMENT.

The laudable Desire of Fame.

When man is contemplated in his connexion with the rest of the species, he is seen to be under the influence of various motives which are peculiar to him in that relation. As long, therefore, as in the transactions of this life, he is the subject of blame or of praise among his fellow-men, he must be furnished by heaven with mental tastes or feelings which correspond to the circumstances in which he is placed. These feelings are generally imparted to every man with such nicety, that the fear of blame and the desire of praise balance each other. It is a base perversion of the original gift, when a man is moved, by the fear of blame alone, to the practice of "what is lovely, fair, and good." And yet, such is the radical defection of human nature, that, in our first attempts to practise what is good, this is generally the preponderating incentive. But when this fear is combined with the desire of

praise, though for a time it acts a subordinate part, it is fulfilling the intentions of Him who bestowed it. Of the two, it is indeed the inferior motive; and when the purposes for which it was communicated are accomplished, it ceases to act any longer as a bias, and gives up the range of its dominion in the mind to be occupied by its former companion,—the desire of commendation. These purposes are fulfilled, when a constantly-virtuous habit of soul is induced, and a man is brought to do that which is lawful and right, under the influence of the desire of praise, and without any respect to the fear of blame.

These principles being established, and it being acknowledged that

"The love of Fame, howe'er subdu'd by art,
"Reigns more or less in every human heart,"

the desire of fame may be considered as another term for *the desire of praise* on an extended scale. To be *laudable*, it must aspire after the approbation of good men, and must be the chief prompter to that which is virtuous. False Fame, ascribed to men for deeds not praiseworthy, is one abuse of this desire; and it is most profusely bestowed on others by persons who entertain no jealousy for the rights of goodness. The desire of being famous in wickedness can never be laudable. Depraved as man is, such a desire is an unnatural feeling; and was never by the Creator implanted in the human breast. By a man's persisting to indulge himself in every variety of vileness, and the seeming approbation of his compeers in turpitude, this itching for adventitious and ill-earned praise is engendered. However untutored in virtue the mind may be, it will always at first naturally recoil at the idea of obtaining such illicit fame as is implied in celebrity acquired by that which is "our nature's greatest infamy." But by repeated acts this native abhorrence is worn down, and the man becomes more dissolute in his living, and yet more abandoned in his character. It is then, to procure the applause of the base, that he commits sin with greediness.

The laudable desire of praise has already been shewn to be the more noble and efficacious principle. This will be more apparent by contrast:—The fear of blame will make a man harmless, by preventing the commission of crime, and at that point its operation often ceases; but it is the peculiar excellence of the desire of praise that it impels a man to the performance of every thing that is virtuous. The former, when alone, is frequently "the minister of sin;" but the laudable desire of fame can never prove itself to be such an emissary. In a mixed community of men, it sometimes happens that the fear of blame will deter a person from that, the performance of which would have been his imperative duty, and the highest proof which he could have evinced of his courage and virtue. Thus the fear of being blamed for such a laudable action, by a number of ignorant or selfish individuals, whose particular interests might be thwarted by it, prevents in many instances the achievement of that which a man is convinced it would be his highest honour to do: And to him that, even in this respect, "knoweth to do good and doth it not,—to him it is sin." But the laudable desire of fame can never thus compromise the interests of virtue, nor desert her righteous cause. Whatever obloquy may for a time attach to a man's strict adherence to the line of duty which his conscience prescribes, he is sure of building on the most durable foundation on which to erect his fame, who, regardless of discouraging circumstances, encounters every opposition and braves all dangers. None but a righteous man can be rightly affected by a laudable desire of fame; and in him it is a passion equally removed from the chill maxims of a worldly policy and from the Quixotic daring of folly and imprudence.

When man is considered in relation to his Maker, as His handy-work, dependent on his bounty, and subject to his laws, he then comes under the obligation of higher motives, and ought to be actuated by the hope of better than earthly rewards. The impracticable ethics of Heathen philosophers intimate that we are to

love and practise virtue for its own sake. But the Divine Author of the christian dispensation knew human nature in its fallen state and all that is in man; and he therefore never requires any thing from him that is not befitting his natural powers and most suitable to his circumstances. His requirements have also this additional recommendation,—that if he claims any service, he graciously communicates sufficient powers to perform it. Let it not therefore be once supposed that the desire of obtaining the approbation of God is an improper inducement, or unworthy the contrivance of Infinite Wisdom. In the sacred records, it is related in commendation of Moses, the servant of God, that '*he had respect to the recompence of reward.*' Since God has designated this '*the recompence of reward,*' it cannot be sinful in man humbly to make use of the same term, and to be animated by the hopes of a like result. This is the highest point to which *the laudable desire of fame* can be carried in this life; and the most sublime enjoyment which it can afford, is the anticipation of that blessed sentence of divine approval, '*Well done, good and faithful servant; enter thou into the joy of thy Lord.*' Every virtuous act performed by a righteous man, may be resolved into a respect to this recompence of reward. The loudest blast from the trumpet of earthly fame, when compared with this stupendous exhibition of God's approval, is as insignificant as is the slender trill of a shepherd's pipe in comparison of the awful and glorious music of thunder. When a righteous man is suffering in his character from the base detractions of crafty and powerful enemies, when he is unjustly confined in a dungeon by demons in the forms of men, or when he is laid on the bed of sickness, this forcible principle alone will support his mind, when the hopes of obtaining a single token of human applause must have entirely ceased.

Laudable therefore it is to desire the praise and approbation of good men; but it is much more commendable to seek after and obtain "the honour which proceeds from God."

JUVENIS.

DE LA POLITESSE.

La politesse est un désir de plaire aux personnes avec qui l'on est obligé de vivre, et de faire en sorte que tout le monde soit content de nous; nos supérieurs, de nos respects; nos égaux, de notre estime; et nos inférieurs, de notre bonté. Enfin elle consiste dans l'attention de plaire et de dire à chacun ce qui lui convient. Elle fait valoir les bonnes qualités; elle leur fait sentir qu'elle reconnoît leur supériorité. Quand vous saurez les élever, ils vous feront valoir à leur tour; ils vous donneront sur les autres la place que vous volez bien leur céder: c'est l'intérêt de leur amour-propre. La plupart du monde ne demande que des manières qui plaisent; mais quand vous ne les avez pas, il faut que vous bonnes qualités doublent. Il faut avoir bien du mérite, pour percer au travers des manières grossières; il faut aussi ne point laisser voir trop d'attention sur vous-même; une personne polie ne trouve jamais le temps de parler de soi.

La politesse est une envie de plaire; la nature la donne, l'éducation et le monde l'augmentent. La politesse est un supplément de la vertu; dans les temps les plus grossiers, où la vertu régnoit davantage, on connoissoit moins la politesse; elle est venue avec la volupté; elle est la fille du luxe et de la délicatesse: on a douté si elle tenoit plus du vice que de la vertu. Sans oser décider, ni la définir, je crois qu'elle est un des plus grands liens de la société, puisqu'elle contribue le plus à la paix: elle est une préparation à la charité, une imitation même de l'humilité. La vraie politesse est modeste; et comme elle cherche à plaire, elle sait que les moyens pour y réussir sont de faire sentir qu'on ne se préfère point aux autres: qu'on leur donne le premier rang dans notre estime. Elle est l'art de concilier avec agrément ce qu'on doit aux autres, et ce qu'on se doit à soi-même; car ces devoirs ont leurs limites, lesquelles passées, c'est flatterie pour les autres, et orgueil pour vous; c'est la qualité la plus séduisante.

La politesse est l'expression ou l'imitation des vertus sociales; c'en est l'expression, si elle est vraie; et l'imi-

tation, si elle est fausse: et les vertus sociales sont celles qui nous rendent utiles et agréables à ceux avec qui nous avons à vivre. Un homme qui les posséderoit toutes, auroit nécessairement la politesse au souverain degré. Mais comment arrive-t-il qu'un homme d'un génie élevé, d'un cœur généreux, d'une justice exacte, manque de politesse, tandis qu'on la trouve dans un homme borné, intéressé et d'une probité suspecte? C'est que le premier manque de quelques qualités sociales, telles que la prudence, la discrétion, la réserve, l'indulgence pour les défauts et les foiblesses des hommes. Une des premières vertus sociales est de tolérer dans les autres ce qu'on doit s'interdire à soi-même; au lieu que le second, sans avoir aucune vertu, a l'art de les imiter toutes. Il sait témoigner du respect à ses supérieurs, de la bonté à ses inférieurs, de l'estime à ses égaux; et les persuader tous qu'il en pense avantageusement, sans avoir aucun des sentimens qu'il imite. On ne les exige pas même adjourd'hui, et l'art de les feindre est ce qui constitue la politesse de nos jours. Cet art est souvent assez ridicule et assez vil pour être donné pour ce qu'il est, c'est-à-dire pour faux.

Les hommes se doivent des égards, puisqu'ils se doivent tous de la reconnoissance. Ils se doivent réciproquement une politesse digne d'eux, faite pour des êtres pensans, et variée par les différens sentimens qui doivent l'inspirer. Ainsi la politesse des grands doit être de l'humanité; celle des inférieurs, de la reconnoissance, si les grands le méritent; celle des égaux, de l'estime et des services mutuels. Le plus malheureux effet de la politesse d'usage est d'enseigner l'art de se passer des vertus qu'elle imite. Qu'on nous inspire dans l'éducation l'humanité et la bienfaisance, nous aurons la politesse, ou nous n'en aurons plus besoin. Si nous n'avons pas celle qui s'annonce par les grâces, nous aurons celle qui annonce l'honnête homme et le citoyen; nous n'aurons pas besoin de recourir à la fausseté. Au lieu d'être artificieux pour plaire, il suffira d'être bon; au lieu d'être faux pour flatter les foiblesses des autres, il suffira d'être indulgent. Ceux avec qui l'on aura de

tels procédés, n'en seront ni enorgueillis ni corrompus; ils n'en seront que reconnoissans, et en deviendront meilleurs.

La politesse consiste encore à se livrer de bonne grâce dans la conversation, à n'avoir pas plus d'esprit que ceux avec qui on se trouve, à n'affecter point trop de justesse, à donner quelquefois lieu à la contradiction et à la critique, en un mot, à n'avoir pas toujours raison. Il ne suffit pas de ne rien dire et de ne rien faire qui puisse blesser les autres; il faut encore souffrir ce qu'ils peuvent dire ou faire d'offensant ou de moins poli. Ainsi une grande partie de la politesse consiste à souffrir l'impolitesse, et à n'y opposer que l'exemple du contraire. Témoigner aux autres qu'ils nous offensent, c'est presque toujours les offenser.

Une hardiesse et une timidité excessives sont également contraires à la vraie politesse, qui veut qu'on parle et qu'on agisse d'un air modeste et d'un air aisé. Quand on ne peut plaire qu'en employant le déguisement, il faut alors renoncer à plaire. Quand la vérité et la politesse sont en concurrence, et qu'on ne peut les accorder entre elles, il faut sacrifier la politesse à la vérité.

Il est bien difficile d'être poli avec ceux qui ne nous plaisent pas; et le malheur des gens d'esprit est que très-peu de gens leur plaisent. Plus on a d'esprit, plus on aperçoit de défauts dans les autres. D'ailleurs les gens d'esprit ont de la peine à s'assujettir à une infinité de petites formalités qui font néanmoins partie de la politesse. Il est donc plus difficile d'être poli, lorsqu'on a beaucoup d'esprit, que lorsqu'on en a que médiocrement. Mais quand un homme d'esprit est poli, il l'est plus qu'un autre, et d'une manière plus agréable. Tout est pour lui une occasion de dire des choses polies; et le tour ingénieux qu'il leur donne, les rend encore plus flatteuses, pourvu néanmoins qu'on ne se défie pas de sa sincérité.

THE CERTAINTY OF DEATH.

A Translation from the French of Charron and Saurin.

The day of Death is the judge of all other days; the time of reaping the fruits of our labours. A good death casts an immortal lustre on the whole course of our life, as an infamous one dishonours it for ever. It was asked Epaminondas, *Whom he esteemed the best, Chabrias, Hippocrates, or himself;* he replied, "Before I answer this question, I must see us all three die." The reason is, we may counterfeit, even to the last moment.

The best use we can make of life is, to learn how to die. We reckon death the worst of evils. Why not await it quietly and patiently, as a thing inevitable, natural, and reasonable? How do we know that death is an evil? No person has ever complained of it. The preachers present us with dreadful views of it, but it is the remorse, the crimes of man, which they exhibit, and not death. Again, why should we fear it? We know not what it is. A wise man said that "to fear death is, to pretend to know that of which all the world is ignorant." We are very unhappy, when we disturb life by the fear of death; and death by the love of life.

What is more natural than death? To be angry at death is, to be angry at being a man. What more foolish than to torment ourselves about an inevitable evil? Do we knock at a door which never opens? What is more inexorable than death? We remedy uncertain evils; and submit ourselves to irremediable ones. In effect, what complaints, what proud murmurs should we not hear against providence, if we were never to die! Every one would curse his lot, if it were never to end. On the other hand, what strange disorders would there be, if some bitterness were not attached to this last moment! We should run to it; and the most trifling vexation would be sufficient to arm us against ourselves.

It is a little thing, not to fear death; there are occasions, where we should know how to despise it. He who does not fear death, is omnipotent. The Emperor Vespasian commanded one Priscus, a Roman senator, not to appear in the senate; or, if he should do so, to refrain from giving his opinion, and threatened to punish his disobedience with death. "Have I not told you," said he, "that I was immortal? You will act as you please, and I as I ought."

To become acquainted with death without fearing it, is the highest degree of perfection to which the human mind is capable of attaining. It is the highest point of felicity which it is susceptible of enjoying in this vale of misery. And this firmness, none but a true Christian is capable of possessing. I say,—*to know* death without *fearing* it;—and it is in the union of these two things, that this effort of the mind, so worthy of emulation, and this perfection of felicity, so capable of bounding our researches and desires, do consist. For, to brave death without being acquainted with it, and to shut our eyes to its terrors, in order to combat it with success, is far from shewing a noble mind; but is only false glory, and the height of misery.

We have seen philosophers able to free themselves from the fear of death, only because they did not know it. They painted it to themselves, annihilating man, making him appear before chimerical tribunals, or enjoy some imaginary felicity. The christian alone can know death without fearing it. He alone can die without regret, remorse, and fear. Religion tranquillises him; he hopes for an eternal happiness, because he has lived, and now dies, a good man; a man, who desires to see God, and to possess him for ever.

A philosopher of the age, whatever strength of mind he may possess, however he may be persuaded of the future annihilation of his soul, will be agitated with a thousand fears at the point of death. He indeed may be able to conceal them from the observation of those who surround his dying bed; but his conscience will not be the less uneasy. We must die christians, in

order to die tranquil; or, at least we must have a religion, which can promise us a recompence after this life. Annihilation is dreadful, even to the wicked.

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JOHN ROBERTS.

SOMNIUM SCIPIONIS.

'Tum rutilus, horribilisque terris, quem Martium dicitis. Deinde subter mediam fere regionem Sol obtinet, 'dux, et princeps, et moderator luminum reliquorum, 'mens mundi, et temperatio, tantâ magnitudine, ut 'cuncta suâ luce illustret, et compleat. Hunc ut 'comites consequuntur, alter Veneris, alter Mercurii 'cursus: in infimoque orbe Luna, radiis solis accensa 'convertitur. Infra autem jam nihil est, nisi mortale 'et caducum, præter animos generi hominum, munere 'Deorum datos. Supra lunam sunt æterna omnia. 'Nam ea, quæ est media, et nona tellus, neque mo-'vetur, et infima est, et in eam feruntur omnia suo 'nutu pondera.'

Quæ cum intuerer stupens, ut me recepi, *Quod? hic,* inquam, *quis est, qui complet aures meas tantus et tam dulcis sonus?* 'Hic est,' inquit ille, 'qui intervallis 'conjunctus imparibus, sed tamen pro ratâ portione dis-'tinctis, impulsu et motu ipsorum orbium conficitur: 'qui acuta cum gravibus temperans, varios æquabiliter 'concentus efficit. Nec enim silentio tanti motus in-'citari possunt, et natura fert, ut extrema ex alterâ 'parte graviter, ex altera autem acute sonent. Quam 'ob causam summus ille cæli stelliferi cursus, cujus 'conversio est concitatior, acuto et excitato movetur 'sono: gravissimo autem hic lunaris atque infimus. Nam 'terra, nona, immobilis manens, imâ sede semper hæret, 'complexa medium mundi locum. Illi autem octo 'cursus in quibus eadem vis est duorum Mercurii 'et Veneris Modorum, septem efficiunt distinctos in-'tervallis sonos: qui numerus rerum omnium fere nodus'

'est. Quod docti homines nervis imitati, atque cantibus, aperuere sibi reditum in hunc locum: sicut alii, 'qui præstantibus ingeniis, in vitâ humanâ, divina 'studia coluerunt. Hoc sonitu oppletæ aures hominum 'obsurduerunt: nec est ullus hebetior sensus in vobis: 'sicut ubi Nilus ad illa, quæ Catadupa nominantur, 'præcipitat ex altissimis montibus, ea gens, quæ illum 'locum accolit, propter magnitudinem sonitus, sensu 'audiendi caret. Hic vero tantus est totius mundi 'incitatissimâ conversione sonitus, ut eum aures hominum capere non possint: sicut intueri solem nequitis adversum, ejusque radiis acies vestra sensusque 'vincitur.' Hæc ego admirans, referebam tamen oculos ad terram identidem.

Tum Africanus, 'Sentio,' inquit, 'te sedem etiam 'nunc hominum ac domum contemplari: quæ si tibi 'parva, ut est, ita videtur; hæc cælestia semper spectato: illa humana contemnito. Tu enim quam celebritatem sermonis hominum, aut quam expetendam 'gloriam consequi potes? Vides habitari in terrâ raris 'et angustis in locis: et in ipsis quasi maculis, ubi 'habitatur, vastas solitudines interjectas: hosque, qui 'incolunt terram, non modo interruptos ita esse, ut 'nihil inter ipsos ab aliis ad alios manare possit: sed 'partim obliquos, partim aversos, partim etiam adversos stare vobis: à quibus expectare gloriam certe nullam potestis. Cernis autem eandem terram, quasi 'quibusdam redimitam et circumdatam cingulis: è quibus duos maxime inter se diversos, et cæli verticibus 'ipsis ex utraque parte subnixos, obriguisse pruinâ 'vides: medium autem illum, et maximum, solis ardore torreri. Duo sunt habitabiles: quorum australis ille, in quo qui insistunt, adversa vobis urgent vestigia, nihil ad vestrum genus. Hic autem alter subjectus aquiloni, quem incolitis, cerne, quam tenui 'vos parte contingat. Omnis enim terra, quæ colitur 'à vobis, angusta verticibus, lateribus latior, parva 'quædam insula est, circumfusa illo mari, quod Atlanticum, quod Magnum, quod Oceanum appellatis 'in terris: qui tamen tanto nomine, quam sit parvus,

'vides. Ex his ipsis cultis notisque terris, num aut 'tuum aut cujusquam nostrum nomen, vel Caucasum 'hunc, quem cernis, transcendere potuit, vel illum 'Gangem transnatare? Quis in reliquis orientis, aut 'obeuntis solis ultimis, aut aquilonis, austrive parti-'bus tuum nomen audiet? Quibus amputatis, cernis 'profecto quantis in angustiis vestra gloria se dilatari 'velit. Ipsi autem, qui de vobis loquuntur, quam diu 'loquentur?

'Quinetiam, si cupiat proles illa futurorum hominum 'deinceps laudes uniuscujusque nostrum à patribus 'acceptas posteris prodere; tamen propter eluviones 'exustionesque terrarum, quas accidere tempore certo 'necesse est non modo non æternam, sed ne diutur-'nam quidem gloriam assequi possumus. Quid au-'tem interest, ab iis, qui postea nascentur, sermonem 'fore de te cum ab iis nullus fuerit, qui ante nati sint, 'qui nec pauciores, et certe meliores fuerunt viri? 'Cum præsertim apud eos ipsos, à quibus audiri nomen 'nostrum potest, nemo unius anni memoriam consequi 'possit: homines enim populariter annum tantummo-'do solis, id est, unius astri reditu metiuntur: cum au-'tem ad idem, unde semel profecta sunt, cuncta astra 'redierint, eandemque totius cæli descriptionem longis 'intervallis retulerint, tum ille vere vertens annus ap-'pellari potest: in quo vix dicere audeo, quam multa 'secula hominum teneantur. Namque, ut olim defi-'cere sol hominibus extinguique visus est, cum Romuli 'animus hæc ipsa in templa penetravit: ita quandoque 'eâdem parte sol, eodemque tempore iterum defecerit, 'tum signis omnibus ad idem principium, stellisque 'revocatis, expletum annum habeto. Hujus quidem 'anni nondum vicesimam partem scito esse conversam. 'Quocirca, si reditum in hunc locum desperaveris, in 'quo omnia sunt magnis et præstantibus viris: quanti 'tandem est ista hominum gloria, quæ pertinere vix 'ad unius anni partem exiguam potest? Igitur alte spe-'ctare si voles, atque hanc sedem, et æternam domum 'contueri, neque te sermonibus vulgi dederis, nec in 'præmiis humanis spem posueris rerum tuarum: suis te

'oportet illecebris ipsa virtus trahat ad verum decus. 'Quid de te alii loquantur, ipsi videant: sed loquen- 'tur tamen. Sermo autem omnis ille, et angustiis 'cingitur iis regionum, quas vides; nec unquam de 'ullo perennis fuit; et obruitur hominum interitu; et 'oblivione posteritatis extinguitur.'

Quæ cum dixisset, Ego vero inquam, *O Africane, si quidem bene meritis de patriâ quasi limes ad cæli aditum patet, quanquam á pueritiâ vestigiis ingressus patriis et tuis, decori vestro non defui: nunc tamen tanto præmio proposito enitar multo vigilantius.* Et ille, 'Tu vero 'enitere, et sic habeto, non esse te mortalem, sed 'corpus hoc. Nec enim tu is es, quem forma ista de- 'clarat: sed mens cujusque, is est quisque; non ea 'figura, quæ digito demonstrari potest. Deum te igi- 'tur scito esse: si quidem Deus est, qui viget, qui sen- 'tit, qui meminit, qui providet, qui tam regit, et mo- 'deratur, et movet id corpus, cui præpositus est, quam 'hunc mundum princeps ille Deus: et ut ipsum mun- 'dum ex quâdam parte mortalem ipse Deus æternus sic 'fragile corpus animus sempiternus movet. Nam quod 'semper movetur, æternum est: quod autem motum 'affert alicui, quodque ipsum agitatur aliunde, quando 'finem habet motus, vivendi finem habeat necesse est. 'Solum igitur, quod sese movet, quia nunquam dese- 'ritur à se, nunquam ne moveri quidem desinit. Quin 'etiam cæteris, quæ moventur, hic fons, hoc princi- 'pium est movendi. Principio autem nulla est origo. 'Nam ex principio oriuntur omnia: ipsum autem nul- 'lâ ex re aliâ nasci potest: nec enim id esset principi- 'um, quod gigneretur aliunde. Quod si nunquam 'oritur, ne occidit quidem unquam. Nam principium 'exstinctum, nec ipsum ab alio renascetur, nec ex se 'aliud creabit: siquidem necesse est à principio oriri 'omnia. Ita fit, ut motus principium ex eo sit, quod 'ipsum à se movetur: id autem nec nasci potest, nec 'mori: vel concidat omne cælum, omnisque natura, et 'consistat necesse est, nec vim ullam nanciscatur, quæ 'à primo impulsu moveatur.

'Cum pateat igitur, æternum id esse quod à seipso 'moveatur, quis est qui hanc naturam animis esse tri- 'butam neget? Inanimatum est enim omne, quod im- 'pulsu agitatur externo: quod autem animal est, id 'motu cietur interiore, et suo. Nam hæc est natura 'propria animi, atque vis. Quæ si est una ex omni- 'bus, quæ sese moveat, neque nata est certe, et æterna 'est. Hanc tu exerce in optimis rebus. Sunt autem 'hæ optimæ curæ, de salute patriæ: quibus agitatus 'et exercitatus animus velocius in hanc sedem et do- 'mum suam pervolabit: idque ocyus faciet, si jam 'tum, cum erit inclusus in corpore, eminebit foras et 'ea, quæ extra erunt, contemplans, quam maxime se 'à corpore abstrahet. Namque eorum animi, qui se 'corporis voluptatibus dediderunt, earumque se quasi 'ministros præbuerunt, impulsuque libidinum volup- 'tatibus obedientium, Deorum et hominum jura viola- 'verunt: corporibus elapsi circum terram ipsam volu- 'tantur; nec hunc in locum, nisi multis exagitati se- 'culis, revertuntur.' Ille discessit: ego somno statim solutus sum."

A Translation from the Latin of Cicero, being the first part of that curious fragment, entitled,

THE DREAM OF SCIPIO.

"When I had come into Africa, a tribune, as you know, to the fourth military legion of M. Manilius, the consul, I desired nothing so much as to meet Masinissa, a king who, for very good reasons, was most friendly to our family. When I approached him, the old man, having embraced me, wept, and looking for some time afterwards up to heaven, said, *O thou sun supreme, and ye other inhabitants of heaven, I thank you, that, before my departure from this life, I behold, in my kingdom, and within this mansion, P. Cornelius Scipio, by whose very name I am comforted. Thus am I ever mindful of that best and most invincible of men.*—Then

we spent that day in mutual inquiries about his kingdom and our republic, and in much conversation on other subjects. When we had been entertained in a royal manner, we continued our discourse a great part of the night, at which time the old man could talk about nothing but Africanus, and remembered, not only all his exploits, but his very sayings.

"After we had retired for the purposes of repose, I, being fatigued with my journey, and having sat up to a very late hour, slept much more soundly than usual.—In my sleep, Africanus appeared to me in a form, which I knew better from his statue than from his person. The causes of this apparition, I believe, was the conversation we had had; for it often happens, that our words and thoughts produce some such effect in our sleep, as that which Ennius describes concerning Homer, of whom he used very frequently to think and speak when he was awake. As soon as I recognised him, I shuddered with terror. But he said, 'Be of good courage, Scipio; cast off all fear, and remember what I shall relate to thee.—Dost thou see that city, which, compelled by me to obey the Romans, is again renewing its ancient wars, and cannot remain at quiet?' At the same time he pointed out Carthage, from a bright and shining station, which was lofty and studded with stars. 'Dost thou see this city which thou, but a raw soldier, art come now to attack? This very city, two years hence when thou art made consul, thou shalt overturn; and that surname shall be acquired by thyself, which thou hast hitherto had by birthright from me. But when thou shalt have destroyed Carthage, shalt have had a triumph decreed to thee, shalt have been censor, and have gone as ambassador to Egypt, Syria, Asia, and Greece; thou shalt in thy absence be chosen consul a second time, and shalt terminate a most terrible war, and shalt extirpate Numantia. But when thou shalt be conveyed in a chariot to the Capitol, thou wilt find the republic disturbed by the measures of my grand-son. Here, Africanus, it will be needful for thee to shew thy

'country the brilliancy of thy mind, thy abilities, and 'wisdom.—But I perceive the course of that time is 'equally as doubtful as that of fate. For, when thy 'age shall have passed through eight times seven re-'volutions of the sun, and those two numbers, (both of 'which from separate causes are accounted perfect,) 'shall have completed for thee the fatal sum by a na-'tural circuit, the whole state will direct itself to thee 'alone, and to thy name: the senate, all good men, 'the allies, the Latins, will turn their eyes to thee: 'thou wilt be the only one in whom the safety of the 'state may be confided. And, in short, being ap-'pointed dictator, thou must support the republic if 'thou wouldst avoid the impious hands of thy rela-'tions.'"

At this when Lælius had cried out, and the rest had expressed more vehement sensations of sorrow, Scipio gently smiling, said, "Pray do not disturb me in my sleep, and in things which have scarcely the semblance of reality; listen to the rest.

"'But Africanus that thou mayst be more ready to 'defend the republic, know this, that for all who have 'preserved, assisted, or augmented their country, a 'place is reserved in heaven where they may enjoy 'eternal happiness. For nothing which can be done 'on earth is more acceptable to that Sovereign God who 'governs this entire universe, than the legitimate union 'of those councils and assemblies of men, which are 'called STATES; the rulers and preservers of which 'return hither, from whence they originally proceeded.' At this part, although I was frightened, not so much by the fear of death as of treachery from my own connexions, yet I asked, whether he and my father Paulus, and others whom we had considered as dead, were still living. 'Yes,' says he, 'they are alive, who have 'fled from the chains of their bodies, as from a prison; 'and indeed, that life of yours, as it is called, is 'death. But observe thy father Paulus approaching 'thee:' At the sight of whom I shed a flood of tears. But he, with embraces and kisses, forbad me to weep.

And I, upon regaining the power of speaking without tears, said, *O most pious and excellent father, pray, since this is life, (as I have just heard Africanus say) why do I remain on earth? Why do I not hasten to come hither to you?* 'Not so,' says he; 'for unless that God 'whose temple all this is which you behold, shall have 'freed you from that prison of the body, there cannot 'be a free passage for you to this place. For men were 'created under the express stipulation to defend that 'globe which you see in the midst of this temple, and 'which is called *the earth;* and a soul was given 'them from those eternal fires, which you call *stars* and '*constellations*, which being globular and round, and 'animated by divine intelligences, complete their cir-'cuits and orbits with astonishing velocity. Where-'fore, Publius, you and all pious men must retain your 'souls in the custody of your bodies; nor must you de-'part from the life of men, without the permission of 'Him who gave it, lest you should seem to avoid the 'lot of humanity, which has been assigned to you by the 'Deity. But do thou, O Scipio, cultivate justice and 'piety, as thy grand-father here, and as I thy father 'have done; the exercise of which, although of great 'importance towards parents and kindred, is yet most 'important towards one's country. Such a life is the 'path to heaven, and to this assembly of the men who 'have already lived, and, who being now liberated 'from their bodies, inhabit that place which you be-'hold.'

"But that place was a circle among blazing luminaries, shining with a most brilliant whiteness, which you, as you have received the term from the Greeks, call *the milky way.* From this spot all other things appeared to me, while contemplating them, most bright and wonderful. But there I beheld those stars which we have never beheld from this part of the earth: and the magnitudes of them all were such as we never supposed; of which that was the least, that shone at the extremity of heaven and nearest to the earth, with a borrowed light. But the starry globes greatly exceed-

ed the magnitude of the earth. At that time the earth itself appeared so small to me, that I regretted the littleness of our empire, with which we touched only a small speck of it.

"When I had more eagerly viewed this earth, Africanus said, 'Pray, how long will thy mind be 'fixed upon the ground? Dost thou not perceive into 'what temples thou hast entered? Thou hast here all 'things connected together by nine globes or rather 'orbs: Of these one is celestial, which being the 'outermost encloses all the rest, and is the supreme 'Deity himself, restraining and confining the others. 'In this are centred those eternal courses of the pla-'nets which have their revolutions. To this, seven, 'having a retrograde motion, are subject, and also the 'sky. Of these, the one which is called SATURN upon 'earth, possesses one globe. Then comes that efful-'gence of JUPITER, both propitious and genial to the 'race of man.'

Eaton-Bishop. B. CROFT GOODISON.

We received other excellent translations from Masters J. Vint, W. Vint, J. Hill, B. Croft Goodison, John Roberts, Henry Emett, W. H. Mollyneux, S. Southell, W. Christian, J. Wellstood.

Dulces ante omnio Musæ. VIRG.

PROLOGUE,

Spoken at Mr. Sigston's Private Concert,

JUNE 17, 1816.

WHERE rival monarchs, potentates, and peers,
And warlike chiefs were wont, in former years,*
Each others' claims with ardent zeal t' oppose,
Their only aim your favour and applause,—
By diff'rent means your cheering smiles we court,
Nor ask the splendid robe, the princely port,

* Previous to this, Mr. S. occasionally entertained his friends with the representation of a drama.

The tragic swell, the mimic hero's rage,
With all the pomp and fopp'ry of the stage:
But little practis'd in such arts as these,
By music's charms alone, we seek to please.
Celestial melody! by heaven design'd
To sooth despair, and raise the abject mind!
When jarring passions agitate the soul,
'Tis thine alone their fury to control.
Descend! and here, fair Goddess, deign awhile
To crown our efforts with thy fav'ring smile,
Then the rapt audience shall confess thy pow'r,
And thund'ring plaudits mark this happy hour.

Queen Square Academy. W. C.

EPILOGUE,

SPOKEN ON THE SAME OCCASION.

O YOU, for whom with trembling hands we try
The secret charms of magic melody!
Whose awful looks or give despair, or bless,
Whose smiles chan cheer us, or whose frowns depress;
Tho' rude and tuneless is our humble strain,
Tho' all our efforts to delight are vain,
Yet let not censure blast our secret joy,
Nor the sweet buds of tender hope destroy.
Unskill'd by magic sounds to melt the soul,
To fire the passions, or their rage control,
No lofty aims our youthful breasts inspire,—
To please our friends our chief, our sole desire;
Then let our youth and inexperience claim
Your fond indulgence, and forbid your blame.

(The King's Anthem heard at a distance.)

But hark! what dulcet notes salute my ear!
A strain to every loyal Briton dear;
With GEORGE's name the warbling anthem swells!
On GEORGE's name the ear with rapture dwells!
Then let us join the pleasing strain and sing
HEAV'N PROSPER BRITAIN, AND LONG LIVE THE KING!

Queen Square Academy. W. C.

MORNING,

WRITTEN IN THE COUNTRY.

SPRING, 1816.

HERE nature makes a grand display;
The sun breaks forth in bright array,
The tuneful lark, with varied lays,
Warbles the God of nature's praise,
The dews, as incense, to the skies
Rich in ambrosial sweets arise,
And from the flow'r-enamell'd ground
Pour health and fragrance all around.
See the Rustic now abroad,
And Pilgrim on his early road;
Rosy Milkmaids too are seen
Tripping lightly o'er the green;
Blithely as they move along,
Loud they chant some love-lorn song,
Echo, from the neighb'ring vale,
Answers to the tender tale.
Nature's charms let me enjoy,
Pleasures that can never cloy!
Let me feel the healthy breeze,
Listen to the murm'ring bees,
Gladsome hear from every bush
The early carols of the thrush,
The cuckoo's note, the cooing dove,
That wake to extacy and love.
Whilst I brush the morning dews,
What a pleasure numbers lose!
Sound refreshing sleep deny'd,
Listless turn'd from side to side,
Every art they try, to keep
Their drowsy frame immers'd in sleep.
Can your slumb'ring bed of care
With the Morning-Walk compare?
With the joys which nature yields?
With the fragrance of the fields?
With the zephyrs of the morn,
Sweeping o'er the waving corn?

Leeds. G. F.

THE SWALLOWS.

In the publication of this paper, it has been the author's endeavour to elucidate a small, but pleasing paragraph, in the page of natural philosophy. "In the swallows, and in all birds that fly much, or that have the most occasion for their wings, it is manifest that their wings are made not only very long, but that they also are placed in the very best part, to balance their bodies in the air, and to give as swift a progression as their wings are capable of; otherwise, they would reel and fly unsteadily. And no less curiously are the feathers placed in the wings, exactly according to their several lengths. The principals set for stay and strength, and these, again, well lined, faced, and guarded with the covers and secondary feathers, to keep the air from passing through; whereby the stronger impulses are made thereupon. Added to this, is the form of their breast, the bone of which is made like a keel, for commodious passage through the air,—to bear the large and strong muscles, which move the wing, and to counterpoise the body in flight."*

The instinct of the swallows has been thought by some to assimilate very nearly to reason, not only as seen in their migration, and in their civil economy, but also in the way in which they build their nests. It is truly wonderful to see them collect and prepare the material, and form their habitations for all the purposes of incubation, with no other implement than that of their little beaks.—An instance of their instinct in building, was lately related to me by a lady in the North.—It was customary for the swallows to build, every year, about the family-hall where she resided. One summer, she observed a pair busily rearing their nest just above one of her windows; the work had been going on for some time; when, behold, just at the period of its completion, the cement, which had attached it to the house, suddenly gave way, and in a moment their little edi-

* Derham.

fice came down in ruins to the ground The disconcerted couple instantly disappeared, and in a short time returned with four more, all of which immediately began afresh their nest, and continued their labour till they had finished it. These journeymen-builders that came to help them, were larger than the swallows, and were supposed by the lady to be swifts.

It was this instinct which brought them to our shores, at the beginning of our summer. Here, when they arrived, they separated and entered upon the various duties of their little families and state; and, after having reared their numerous offspring, and performed the purposes of heaven, it is this instinct which assembles them together in such a vast body; which reveals to them the near approach of winter with all his dreary and destructive train; and which teaches them to escape, by a timely flight, to a region more congenial to their nature, and more friendly to their existence. The stork in the heavens knoweth her appointed time, and the turtle, the crane, and the swallow, observe the time of their coming:—Instinct alone leads these birds to provide for the evil day. But careless and inconsiderate man, who is endowed with reason, conscience, and all the faculties of an immortal mind, makes no preparation for the hour of death and for the day of judgment!

These birds are considered by many persons as useless in creation, or, at the best, as only an ornamental part of it; but this is altogether a mistaken idea. It is ascertained by naturalists, that the swallow and the whole race of *hirundines*, take their food only while on the wing; consequently, they must live chiefly upon the small winged insects, such as gnats, flies, and moths, which crowd the air in summer; the destruction, therefore, of these insects, is not to be considered lightly—it is an universal good. Flies were one of the plagues brought upon Egypt, and hornets one of the means whereby the Canaanites were driven out of their land; and, at the present time, large tracts of country in the South Seas, are rendered totally uninhabitable by the immense swarms of flies and moschet-

tos which infest them. We may not, it is probable, anticipate much evil from the existence and increase of these injurious creatures here; yet were it not for the kindly aid of the swallows, the numerous insects which are natural to our climate, might so far multiply, as to make our healthy and beautiful country at times highly distressing, if not pestilential. How wicked is the conduct of those, who cruelly harm or destroy these useful birds! It is an injury done to our country, as well as a violation of the laws of nature and a sin against God. If, as a judicious and respectable natural historian has written, a tame and young swallow would eat from seven hundred to a thousand flies in one day, what an infinite destruction of noxious insects must an army like unto this have made. Considering them in this light, what a blessing are these birds to our land!—It is this that causes me to regard them at this time, as God's host, returning from the accomplishment of his merciful and gracious designs towards us. The swallows also are, doubtless, intended to beautify nature, and to increase the happiness of man, by giving him pleasure, as well as to be profitable to him. When God created the earth, although it was perfect, and contained every thing necessary for the support of man; yet it was not considered a fit habitation for him, until God had filled it with living creatures: these, by giving life and beauty to the woods and plains, diffuse a pleasant feeling to the heart of man, who by nature is formed for society. I very well remember, when only as yet a boy at school, viewing with astonishment and delight their grave assemblage and their sage debate, when, preparatory to their flight, they had covered the roof of our school-room. And now age has sobered reason, and tempered the ardour of youth, I still retain the same fond feelings for them. For my part, I cannot spare them in my summer-evening walks. I love to stand and watch their airy sports and dextrous flight.—I love to see them sudden dart across the pond, dipping their dappled wings, and catching the floating flies.—I love to mark their skill and industry in build-

ing, and watchful care in rearing their brood,—and even their chattering, at the break of day, down the chimney of my chamber, gives me a pleasure. All tend to calm the mind, and to instil peace. O that the sight and company of man always created the same feeling!

Early in the month of September, 1815, the swallows, that beautiful and social tribe of the feathered race began to assemble in the neighbourhood of Rotherham, at the Willow Ground, near the Glass-house, on the banks of the canal, preparatory to their migration to a warmer climate; and their numbers were daily augmented, until they became a vast flock, which no man could easily number—thousands upon thousands, tens of thousands, and millions—so great, indeed, that the spectator would almost have concluded, that the whole of the swallow race were there collected in one huge host. It was their manner, while there, to rise from the willows in the morning, a little before six o'clock, when their thick columns literally darkened the sky. Their divisions were then into four, five, and sometimes into six grand wings, each of these filing off and taking a different route—one East, another West, another South, and so on;—as if not only to be equally dispersed throughout the country to provide for their numerous troops; but also to collect with them whatever might be left behind of their straggling fellows, or parties. Just before the respective columns arose, a few birds might be observed first in motion, at different points, darting through their massy ranks;—these appeared like officers giving the word of command.

In the evening, about five oclock, they began to return to their station, and continued coming in from all quarters, until nearly dark. It was here that you might see them go through their various aërial evolutions, in many a sportive ring and airy gambol—strengthening their pinions in these playful feats, for their long etherial journey; while contentment and cheerfulness reigned in every breast, and was expressed in their evening song by a thousand pleasing twitters from their little throats, as they cut the air and frolicked in the last

beams of the setting sun, or lightly skimmed the surface of the glassy pool:—the notes of those that had already gained the willows, sounded like the murmur of a distant waterfall, or the dying roar of the retreating billow on the sea beach.

I also observed in the swallows the strictest regard to *order*. In the morning, at the stated time, they arose from their leafy beds; and though their numbers were so immense, yet there was not the least confusion. It was their general custom to rise in divisions, and that in such regular time and manner, as prevented them from mistaking the one wing for the other, or pressing the one upon the other,—each bird seemed to know its own exact place, company, and division. When they returned in the evening, some advanced first, as if to give notice of their approach; and soon were seen the respective subdivisions, divisions, and columns, arriving at different periods to their willow encampment. They did not immediately, on their arrival, *all* alight; this would have caused disorder and mischief; but while the main body was performing its evening exercises, companies were continually filing off to their quarters. This not only prevented accident in their descent, but the column which remained in the air served as a beacon for those whose greater distance of operation might have prevented their return before the evening shades came on.

The verdant enamel of summer had already given place to the warm and mellow tints of autumn, and the leaves were now fast falling from their branches, while the naked tops of many of the trees appeared—the golden sheaves were safely lodged in the barns, and the reapers had, for this year, shouted their harvest home—frosty and misty mornings now succeeded, the certain presages of the approach of winter. These omens were understood by the swallows as the order for their march: accordingly on the morning of the 7th of October, their mighty army broke up their encampment debouched from their retreat, and, rising, covered the heavens with their legions; thence, directed by

an unerring guide, they took their trackless way. On the morning of their departure, when they ascended from their temporary abode, they did not, as they had been wont to do, divide into different columns, and take each a different route, but went off in one vast body, bearing to the south. It is said that they would have gone sooner, but for a contrary wind, which had for some time prevailed: that on the day before they took their departure, the wind got round, and they immediately embraced the favourable breeze. England, it appeared, had served the purposes for which it was designed for them; and now her chalky cliffs, her sylvan shades, her fertile plains, her peaceful cities, and her healthful villages, had lost their charms—and nothing that she possessed could tempt them back.

On the day of their flight, they left behind them about a hundred of their companions: whether these were slumberers in the camp, and so had missed the going of their troops, or whether they were left behind as the rear-guard, it is not easy to ascertain: they remained however, till the next morning, when the greater part of them mounted on their fleet pinions, to follow, as it should seem, the celestial route of their departed legions. After these, a few stragglers only were seen; these might be the sick or too young to attempt so great an expedition;—whether this was the fact or not, they did not remain after the next day. If they did not follow their army, yet the dreary appearance of their depopulated camp, and their affection for their kindred, might influence them to attempt it, or to explore a warmer and a safer retreat.

Much has been said, at different times, concerning the migration of the swallows. It is true, that now and then a few birds have been discovered in the winter, laid up in some crevice or cave. It is probable that they have been of the description of these last remaining, which were supposed to be the sick or late-hatched birds. But to imagine that the many thousands which were here met together, should have no other object in view but that of diving into ponds, or of lying up

in some cleft,—is an idea that a child in philosophy would be able to confute. Nor can the general question for their migration be the least affected, by the remaining of these few birds behind. It would be like maintaining, that man was a creature that never lived out of a house, because some children or sick persons should be confined to their room.

The fact related by Sir Charles Wager, clearly establishes the migration of these birds:—"Returning home," he says, "in the spring of the year, as I came to sounding, in our Channel, a great flock of swallows came and settled on all my rigging; every rope was covered; they hung on one another like a swarm of bees; the decks and carving were filled with them. They seemed almost famished and spent, and were only feathers and bones; but being recruited with a night's rest, took their flight in the morning." In Kalm's voyage to America, is another instance to the point; for a swallow lighted on the ship in which he was sailing, on the second of September, when he had passed over two-thirds of the Atlantic Ocean.

After contemplating the wisdom and goodness of God in these his works, where is the soul that can refrain from the celebration of his praises, and that will not unite in the language of holy David, and say, "The Lord is good to all, and his tender mercies are over all his works. All thy works shall praise thee, O Lord, and all thy saints shall bless thee. My mouth shall speak the praises of the Lord, and let all flesh bless his holy name, for ever and ever!"

*** These particulars relating to the swallow tribe, are extracted from a very interesting pamphlet, written by a clergyman, and entitled, *The Swallows, or Observations and Reflections on their late Assemblage at Rotherham, and their subsequent departure.*

QUERIES.

(1.) *By* Mr. J. BURLEY.

WHAT are the different parts of speech of the word "*to*," in the following sentences, taken from Mr. Murray's "few instances of the same word's constituting several of the parts of speech:"——*They strive* TO *learn. He goes* TO *and fro'. To his wisdom we owe our privilege. The proportion is ten* TO *one.*

(2.) *By* Mr. ROBERT MAFFETT, *Plymouth.*

Have the tails of comets any solid nucleus?

(3.) *By* J. F. THORP-ARCH.

Does the Imperative, in the English language, really admit of a first person singular?

(4.) *By the same.*

What is the reason of the apparently-increased magnitude of the discs of the sun and moon at the time of their rising and setting?

(5.) *By* Mr. JONAH SNIBE.

In the business of teaching, whether is it more advantageous to begin with the practical or with the theoretical parts of science?

(6.) *By* ANASTHIMENES.

It is generally acknowledged that Beer or Ale brewed by Publicans or licensed Brewers is of a more stupifying and intoxicating quality than that which is home-brewed. What is the reason of this difference?

(7.) *By* Mr. JOHN BAINES, *Nottingham.*

What is the cause of that extraordinary whirlpool, called Maelstrom, on the coast of Norway?

ANSWERS TO THE QUERIES.

Which were proposed in the Second Number of the Second Volume.

Answers to the Queries were severally received from Messrs. J. Baines, W. Shenstone, J. Brooke, Jonah Snibe, T. Pilkington, J. Hirst, and from Anasthimenes, Quæsitor, Grammaticus, and Investigator.

(1) On the expansion of Water by Frost.

By ANASTHIMENES.

In winter when the temperature of the atmosphere is reduced so as to give the water a tendency to freezing, those particles of water on the surface, which are first cooled, acquire a greater specific gravity; and, sinking down, warmer particles ascend to supply their place, and are cooled in their turn and descend. When this circulation ceases, it might be supposed that, as the cold increases, the density of the freezing water would continue to increase; and that the ice, as soon as formed, would be precipitated. But here nature, all bountiful, has made a memorable deviation: For no sooner is the water cooled down to 40°, than it ceases to increase in density, and, on the contrary, actually becomes of less specific gravity as the caloric is further abstracted by the cold atmosphere.

When this diminution of specific gravity has begun to take place, the water continues enlarging its bulk in regular gradation till the surface becomes cooled down to 32°, when a film of ice forms and covers the whole mass. This seems to be the reason; though it is observable that ice is formed in hexagons and different other bodies.

By Mr. JOHN BAINES.

Experiments made by the Academy del Cimento prove beyond a doubt that frozen water occupies more space than fluid water. Bomb-shells and the strongest vessels, being filled with water, were burst into pieces

by the congelation of the fluid. The trunks of trees have been split and divided with a loud noise, by the freezing of the sap which they contained; and stones have been broken in pieces, the moment the water with which they were impregnated passed into a state of ice. But these effects are probably not to be ascribed to the water, but to the air which is disengaged during the time of congelation; nevertheless, it is evident, the specific gravity of ice is less than that of water, on which it swims; its formation is probably a confused chrystallization, occasioning it to contain a greater quantity of air in proportion to its bulk. For if the cavities of ice be opened under the water, the air which they contain will be seen to issue in bubbles from its surface. Bodies therefore experience a dilatation proportional to the accumulation of caloric, when they suffer no change either in combination or the quantity of chemical principles; but this law does not hold good when those changes take place.

(2) Is coal a semi-metal?

By Mr. J. BAINES.

Coal is, with more probability, a vegetable substance, than a semi-metal; because it forms the residuum of all vegetable substances that have undergone a slow and smothered combustion; and of all putrid animal bodies. Water which has passed through a putrid dunghill, is of a brown colour; and if it be subjected to evaporation, the principal part of the residuum will be found to consist of coal. Coal is found in fixed air, of which it constitutes nearly one fourth part. We are indebted to M. Hassenfratz for the discovery that it is an essential ingredient in the food of plants, and next to water the most abundant in all vegetables. Wood and corn contain the most, and grasses the least quantity of coal. Wiegleb found dry beech wood to contain one fifth part of its weight, and Westrumb discovered that a kind of clover (the *Trifolium pratense*) contained one seventh. In the process of paring and burning, the oleaginous and mucilaginous particles of the soil are re-

duced to a coal, and the fixed air combined with the earths is set loose by various processes, and absorbed by the soil; and hence we are enabled to form an idea why soot is such a powerful manure. The chemical properties of pit-coal, are,—by distillation it affords ammoniac, which is dissolved in water while the oil floats above. When it is deprived, by a slow combustion, of all its oil and volatile particles, the earthly residue contains Sulphates of Alumine, Iron, Magnesia, and Lime. When the combustion is rapid, the sulphur is dissipated, and there only remains the aluminous, magnesian, calcareous, and other earths, but the alumine generally predominates. Petroleum, Naphtha, Mineral Pitch, and Asphaltes, are slight modifications of bituminous oil, so abundant in pit-coal. Petroleum is generally found in the vicinity of coal-mines, and near volcanoes; the heat of the decomposition of the pyrites contained in this substance being sufficient to disengage bituminous oil from the coal. Naphtha, or pure rock oil, is found in great abundance on the western coast of the Caspian Sea, near Baku. The earth when dug to the depth of two or three inches, will easily take fire. (See Vol. 1st. Page 186.)

(3.) On the rays of the sun passing through glass.

By PYRODES.

If we allow the present theory of the sun to be correct,* we shall easily account for the effect that is here produced. For if the rays of the sun do not contain heat, but only produce it by the force with which they strike upon bodies, they cannot produce heat in transparent bodies, because they will pass freely through them. Therefore they will produce heat by falling upon opaque bodies after they have passed through the glass, though the glass remains cold.

(4.) Which is the strongest of the passions?

By Mr. JONAH SNIBE.

All the passions by which the mind is agitated, spring from two grand fountains, viz. ANGER and LOVE.

* See the eighth query.

The first is the parent of malice, hatred, envy, disdain, jealousy, fear, &c. and the second of friendship, hope, melancholy, joy, &c. But it will readily be granted that no branch of a stream is as strong as the parent stream from which it proceeds. Hence, we have only to determine whether Anger or Love be the stronger; and for this purpose nothing will furnish stronger proofs than the daily occurrences of life. Anger is observed to be temporary, particularly in its most violent effects. It sometimes occasions death, and that even in a very precipitate manner. When its effects are not so violent, a fever is the consequence; but, unless the fit of anger be renewed, the fever quickly subsides. Love, on the contrary, is slow in its effects. It sometimes originates quickly, but is more generally a great length of time in arriving at maturity; when, if the object beloved be not obtained, symptoms of the most alarming kind are observed in the love-sick person. Loss of appetite, restlessness, and anxiety of mind are accompanied by a slow fever, which together seldom fail, in a very short time, to put a period to the life of the person. There are several instances on record, in which the possession of the object of love could not stop the progress of the disease after it had arrived at a certain stage. The most remarkable incident of the kind, is the true story of "Edwin and Emma," recorded by Mallet. The parties were both buried in one grave, in the church-yard of Bowes, Yorkshire, and the curate inserted in the register, that they "died of Love." From what has been said, it will plainly appear, that ANGER is the most violent passion, but LOVE is the most durable.

(5) What will be the best substitute for "he used?"

By Mr. J. BROOKE.

I think, "he was accustomed," or, "he usually did, &c." will appear as well; though it seems to me, "he used" is very intelligible; and, I think, not improper.

By GRAMMATICUS.

He used, is a phrase neither improper nor inelegant. But as a repetition of the same words is not pleasing,

and as variation in expression without weakening the sense is very desirable in composition, *he was wont*, which is a fine old English phrase, would be a very good substitute. *It was his manner or humour, This was his constant practice. He accustomed himself to act, &c.* would all be appropriate expressions and would intelligibly convey the meaning proposed.

(6) On the means by which flies walk along the ceiling of a room with their bodies downwards.

By Mr. T. PILKINGTON.

For an answer to this query we are indebted to Sir Everard Home and Sir Joseph Banks; to the former, for his anatomical observations; and to the latter, for his information to Sir Everard Home, of an animal of similar habits to those of the fly, as far as relates to the present point of consideration. It has been generally supposed, that flies have supported, or, rather, suspended themselves, when in a position opposite to gravity, by sharp claws, with which their feet have been thought to be furnished. But I think those who have been favourers of the claw-system, will see ample reason to change their opinion from the following facts which I shall adduce.

Sir Joseph Banks, while resident at Batavia, in the island of Java, noticed an animal, the *Lacerta Gecko*, a native of that place, which can walk the polished surface of a perpendicular plane. This animal, in the evening, will descend from the roofs of the inhabited houses in quest of its prey, walk down the hard, smooth, polished walls of Chucam, seize upon any flies which may have settled upon them, and then retreat, by running up to the roof, his place of safety. Sir Joseph frequently caught these creatures in the following manner: he procured a long flattened pole, and therewith scraped suddenly along the face of the wall, and thus knocked the animal down. He furnished Sir Everard Home with one of these animals of a very large size, whose weight was five ounces three quarters avoirdupoise. By a careful examination of the feet of the *Lacerta Gecko*, Sir Everard was enabled to discover the

means by which it can maintain its hold with such ease upon a smooth and upright surface, notwithstanding the considerable weight of its body.

He states that the anatomical structure of this animal's foot is such, that it has the power of forming a number of concavities resembling little inverted cups. Thus, it forms, when walking upon surfaces as stated above, *a vacuum* in these concavities, between the plane it walks upon and its foot. Hence, by the pressure of the atmosphere, the *Gecko* is maintained in its position. Sir Everard, having thus discovered the principle upon which this animal attaches itself so firmly to places so unfavourable to progressive motion, turned his attention to the common fly, which supports itself in situations still more disadvantageous than what are already enumerated of the *Lacerto Gecko.* He found by careful examination, that the fly's foot is constructed in a similar manner, and is capable of producing concavities by which it attaches itself to, and walks along the ceiling of a room, the under side of glass globes and other bodies, in all directions.

Therefore, it now appears that the attachment of the Lacerto Gecko and the fly, to a plane opposite to gravity, is effected by means of atmospheric pressure. Many other little animals, or insects, of similar habits, will be found, upon due observation, to support themselves in a like manner.

(7.) On the Style of Addison and Johnson.

To the Editor,

Sir,—Fully to execute the task which your Correspondent Investigator has assigned to his friends, in your last number, would require a greater compass of mind than that which a man of common abilities and of ordinary modesty, would allow himself to possess. For, in the variety of their styles, literary men are influenced by their own turn of thought, mode of conception, and habit of combination, as well as by their partialities for the style of their favourite authors. To prevent the operation of this contagion,

is no easy matter. It will betray itself even in professed imitations. Consequently, a great difficulty will always arise in exhibiting the distinct features, and embodying the manner of other men's compositions.—The task proposed also implies that the man who undertakes it, should have an intimate and enlarged acquaintance with the most chaste and classical productions of British genius, which, for the honour of my native country, I am happy to say, would occupy the whole life of a learned man, and, by its immense stores, such as no other language in the world can furnish, would amply repay him for his most fascinating labour and delighted attention.

Yet one portion of the appointed labour will be best performed by a person endued with a mediocrity of intellect, and who has a knack at mimickry and imitation. Such a writer will produce a greater resemblance to the style of another his superior, than can be done by a man of sterling genius. You are shrewd enough to perceive from this preamble, that if I fail in a correct delineation of the characteristic mode of expression, and flow of language, employed by Addison and Johnson, the very failure will, according to these premises, be a capital proof of my learning and talents. Under this consideration, to the execution of the plan I may joyfully proceed.

While the style of Addison is universally acknowledged to be simple, pure, and elegant, its distinguishing traits are that it is graceful yet not affected, correct without stiffness, intelligible yet not colloquial, and rich without exuberance.

Johnson's style is distinguished for copiousness, majesty, strength, and melody. He had the peculiar felicity, to which no preceding writer had attained, of combining sweetness with dignity of style, and strength with harmony.

Johnson is generally nervous, Addison equally energetic: Yet a few sentences might be selected out of the polished writings of Johnson, in which it will be perceived that, from too great a degree of tension, a

correspondent degree of feebleness has ensued. And in Addison, some sentences, still fewer in number, will be found, which are feeble from another cause, from their being so chaste as to chill, and possessing such smoothness as to produce no friction on the mind, and thus losing the effect they were intended to produce.

(The remaining part of our correspondent's paper, and others on this subject, must be reserved for the following numbers of the work.)

(8) Is the sun a great globe of fire?

By PYRODES.

Of all the theories of the sun which have been offered to the philosophical world, that of Dr. Herschel's appears to be the most rational. It is briefly as follows: The Doctor considers the sun to be the most magnificent habitable globe in our system, surrounded by a double set of clouds. Those which are nearest its opaque body, are less bright and more closely connected together, than those of the upper stratum, which form the luminous apparent globe we behold. This luminous external matter, he observes, is neither a liquid nor an elastic fluid, of an atmospheric nature; for, in either of those two cases, it would not admit of any chasms or openings. Therefore it must be concluded that this shining matter exists in the manner of luminous clouds of an electrical nature, only producing light; and that the various spots that we see on the surface of the sun, are owing to accidental openings in this luminous atmosphere, through which we see the more opaque clouds, or the dark body of the sun beneath; hence, the spots appear of different shades. I refer those who wish to see more upon this theory of the sun, to some beautiful remarks made on it, from page 533 to page 537, of Smith's Panorama of Arts.

I shall, in the next place, endeavour to prove the truth of this theory, from experiment. Heat is always produced by any means that violently agitate the particles of bodies, as by friction, percussion, and the like. For we see that the hammering of a piece of iron smartly upon an anvil, is capable of rendering it red-

hot. And the collision of flint and steel, is capable of producing a sufficient degree of heat to fuse the particles that are struck off. No wonder then that the particles of light, flying with the incredible velocity of 164000 miles in a second of time, (though they be almost infinitely small,) should be capable, with the force with which they will strike upon opaque bodies, of producing that degree of heat that they are known to produce.—In the first place, to prove from experiment, that the rays of the sun do not contain heat: If they be condensed by means of a burning glass, till the heat produced is capable of fusing iron, and a decanter of clear water be placed in the focus, the water will not be warmed. For, in this case, the rays pass freely through it, and have nothing upon which to exert their force. But if the water be made opaque, by means of a little ink, so that the light has something to impinge upon, the water will quickly be rendered hot. Again, if a piece of wood be fixed to the bottom of a glass, and the glass be filled with water, and exposed to the *focus* for some time, so that the rays may fall upon the piece of wood, no apparent change will take place: But if the wood be taken out of the water and broken, the inside will be found burnt to a coal. This experiment proves that the heat that burnt the wood, could not come from the sun, but must be contained in the wood itself; otherwise the outside of the wood must have been burnt, and the water rendered hot. I could here bring many more experiments to prove the truth of this theory, such as the effects of the rays of the sun on the tops of mountains, and the effect that is felt when a person ascends in a balloon. But as they would swell the account too much, I must be content with what I have already stated.

By Mr. JOB SHAINEN.

Various opinions have been formed by philosophers respecting the nature of the sun. Anaximander imagined that there was a circle of fire encompassing the heavens, between which and the earth there was ano-

ther circle of some kind of opaque matter. In this latter circle there was a hole, through which the light was transmitted, and appeared to the inhabitants of the earth like a round and distinct body of fire. Anaxagoras supposed the sun to be a fiery globe of some solid substance; and Sir Isaac Newton proposed it as a query, "Whether the sun and fixed stars were not "great globes vehemently hot, whose parts were pre-"vented from fuming away by the great weight and "density of their surrounding atmospheres." From the known circumstance of spots being observed on the sun's disc, some have thought it to be an opaque body, mountainous and uneven, and covered all over with a luminous fiery substance, which, by the rotation of the sun on its axis, is subject to great irregularity, so as sometimes to leave the tops of the most elevated places bare, which appear like so many black spots, the nebulosities about them being caused by little cavities filled with the luminous fluid. Others have conceived the sun to be formed entirely of a fiery luminous fluid, wherein are immersed opaque bodies, which by the rapid motion of the sun are sometimes brought to the surface, where they form spots, and change their shapes as different sides of them become visible.

These were the opinions of the ancient and some of the modern theorists: but since the solar rays have been analyzed by Dr. Herschel, philosophers have begun to imagine that the sun is not at all a hot body. It is now known to emit two different kinds of rays, one which illuminates or produces vision, and the other which excites heat by coming in contact with, or diffusing itself through, the atmosphere of the earth. It is also known that the quantity of heat generated by the presence of the calorific rays, is greater in the denser than the rarer regions of the atmosphere: insomuch, that on the summits of very elevated mountains, owing to the extreme rarity of the atmosphere, eternal frost prevails. This is corroborated by aerial voyagers, who find the cold to increase as they ascend above the earth. Hence it is as reasonable to suppose the sun to be an im-

mense globe of ice or snow, as that it is a body of liquid fire. But all that can be said about it, is mere conjecture and probability; we can only reason from analogy: and in this case we have a very forcible argument in support of the sun's being composed of cold matter, viz. if cold water be poured upon quick-lime, sensible heat is generated; and if we imagine beings to exist in the quick-lime, it is a thousand to one that they would believe the water to be hot; but we know the contrary to be the case.

ANSWERS

TO THE

QUESTIONS FOR YOUTH,

Proposed in the Second Number of the Second Volume.

(1) By Master J. Aleon, *Sheffield.*

There are two numbers, the one 45 and the other $\frac{1}{3}$ as much. Find the product of their squares and the difference of their product and sum.

Answered by Masters Frederick Fleck *and* W. Dyson.

First $45^2 \times \left(\frac{45}{3}\right)^2 = 455625$ = product of their squares, and $45 \times 15 - (45 + 15) = 615$ = diff. of their prod. and sum.

(2) By Master T. Morton.

Two travellers A and B set out at the same time, A from London to Canterbury, going 4 miles the first day, 8 the second, 16 the third, and so on. B from Canterbury to London travelling 6 miles the first day, 12 the second, and 24 the third, and so on. Whereabouts on the road will they meet?

Answered by Master J. VINT, *Idle.*

Let a be the distance between London and Canterbury, and x the miles A will go before he meets B who will consequently go $a - x$ miles Now as the geometrical ratio in both progressions is the same, we have the following proportion $x : a - x :: 4 : 6$, or $x = \frac{2a}{5}$ the distance of the place where they will meet from London.

The same by Master WM. DAVIS.

It is evident that the distance travelled by A is to the distance travelled by B in the same time, as 2 to 3. Therefore taking the distance from London to Canterbury at 56 miles we have

As 5 $\left\{\begin{array}{l} 2 : 22\frac{2}{5} \text{ miles, A's dist. from London,} \\ 3 : 33\frac{3}{5} \text{ —— B's dist. from Canterbury,} \end{array}\right\}$ when they meet. :56::

(3) By Master H. METCALF.

A gentleman has a square plantation whose diagonal is just 3 chains longer than its side. Required its area.

Answered by Masters BLAKEY MARSHALL, *and* C. WATERHOUSE.

Let x = side of the square, then by Euc. 1.47, $2x^2 = (x + 3)^2 = x^2 + 6x + 9$, or $x^2 - 6x = 9$; hence $x = 3 + \sqrt{18} = 7.24264$ ∴ the area is 52.455844 chains.

The same by Masters J. GREENHOUGH *and* C. THOMAS.

Put x = the side of the square, then by the question and Euc. 47.1 we have $x\sqrt{2} = x + 3$, or $x\sqrt{2} - x = 3$ ∴ $x = \frac{3}{\sqrt{2} - 1} = 3 \times (\sqrt{2} + 1)$ and $x^2 = 9 \times (3 + 2\sqrt{2}) = 52.455844$ chains, the area required.

(4) By Master A. WASHINGTON.

A and B played at marbles, B won $\frac{3}{4}$ of what A had, and then lost $\frac{1}{4}$ of his stock, when it appeared that B had only $\frac{2}{3}$ as many marbles as A; but when A gave him 10 they had each the same number. How many had each at the first?

Answered by Masters J. BOTTERILL, H. METCALF, *and* W. VINT.

Let x = what A had, and y = what B had at first. Then we find by the question that when they had discontinued their play A had $\frac{x}{4} + \frac{y}{2} + \frac{3}{8}x$, and B had $\frac{y}{2} + \frac{3}{8}x$, but then it appears that $\frac{2}{3} \times \left(\frac{x}{4} + \frac{y}{2} + \frac{3}{8}x\right)$, or $\frac{y}{3} + \frac{5x}{12} = \frac{y}{2} + \frac{3}{8}x$, whence $x = 4y$. Again, because when A gave B 10 marbles they had each the same number, therefore $\frac{x}{4} + \frac{y}{2} + \frac{3}{8}x = \frac{y}{2} + \frac{3}{8}x + 20$, and by reduction $x = 80$, and consequently $y = 20$.

(5) By Master R. WEST.

The sum of the solidity and superficies of a cube is just 100. Required its side.

Answered by Master J. BURLEY.

Let x = side of the cube required, then by the question $x^3 + 6x^2 = 100$. Now to take away the second term let $x = y - 2$, then $x^3 = y^3 - 6y^2 + 12y - 8$ and $6x^2 = +6y^2 - 24y + 24$ $\Big\} = 100$.

Whence by addition $y^3 - 12y = 84$

Substitute for a and b, in Cardan's formula, the numbers 12 and 84 respectively, and $y = \sqrt[3]{\frac{1}{2}b + (\frac{1}{4}b^2 + \frac{1}{27}a^3)^{\frac{1}{2}}} - \frac{\frac{1}{3}a}{\sqrt[3]{\frac{1}{2}b + (\frac{1}{4}b^2 + \frac{1}{27}a^3)^{\frac{1}{2}}}} = 5.2822$; hence $x = 3.2822$.

The same by Masters W. GODWARD, R. LORAINE, C. MACANN, *and* C. SAUNDERS.

Put x = the side of the cube required, then per question $x^3 + 6x^2 = 100$; whence by trial and error, $x = 3.28226$.

(6) By Master R. NECK.

Two persons A and B travelling together, A with 100£. and B with 48£. met a company of robbers, who took twice as much from A as from B, leaving A 6 times as much as they left B. What did they take from each?

Answered by Masters J. ABBOTT, H. ALLEN, WM. KITSON, *and* J. STEPHENSON.

Let $2x$ and x denote the number of pounds taken from A and B respectively, then per question $100 - 2x = 3 \times (48 - x) = 144 - 3x$; whence $x = 44$£. taken from B, and $2x = 88$£. from A.

The same otherwise by Masters J. ALGOR, J. BURLEY, JARRATT, MACANN, *and* M. SANDERSON.

Suppose the robbers took 2£. from A and 1£. from B; then $(48 - 1) \times 3 - (100 - 2) = 43$, the first error.

Again, suppose they took 4£. from A, and 2£. from B, then $(48 - 2) \times 3 - (100 - 2) = 42$, the second error. Whence by the rule of Double Position $\frac{4 \times 43 - 2 \times 42}{43 - 42} = 88$£. = what they took from A, and $88 \div 2 = 44$£. they took from B.

(7) By Master T. CUVELJE.

Admit a person to ascend 2 miles above the surface of the earth, how many square miles will be visible to him, the earth being a sphere whose radius is 4000 miles?

Answered by Miss M. WEST, *and* Master J. ROBERTS.

By the property of the circle as 4002 : 4000 :: 2 : 1.9990005 = versed sine of the visible spherical segment to an eye 2 miles above the earth's surface: then by Mensuration $8000 \times 3.1416 \times 1.9990005 = 50240.4798$ square miles, the answer.

(8) By QUÆSITOR, *Manchester.*

If to my age you 30 add,
And triple the amount,
Then take the square root of the same,
Two thirds my age you count.

Answered by Masters J. GODWARD, B. C. GOODISON, *and* THOMAS SAMPSON.

Let x = his age, then per question $\sqrt{3x+90} = \frac{2x}{3}$; whence $x^2 - \frac{27x}{4} = \frac{810}{4}$: this equation solved gives $x = 18$ years, the answer.

Ingenious answers to all the questions were given by Masters J. Roberts, *Mr. J. H. Swale's Academy, Brunswick-Place, Liverpool;* Humphrey Allen, *Rev. H. Davis's Seminary, Eaton Bishop;* John Stephenson, *Academy, Kilham;* John Abbott, *Brindle;* Wm. Kitson, *Mr. H. Lee's Academy, Windhill, Bradford;* Joseph Greenhough, *Bolton, near Bradford;* Thomas Sampson, *Mr. Hudson's Academy, Epworth;* Wm. Davis and Matthew Sanderson, *Mr. Jaques' Academy, Penrith;* Wm. Jarratt, *Academy, Kilham;* John Vint, *Rev. Mr. Vint's Academy, Idle;* Blakey Marshall, and Charles Waterhouse, *Mr. Garside's Academy, Mirfield;* J. Burley, *Mr. Brooke's Academy, Wortley;* Miss Mary West, *Haxey, near Gainsbro';* Charles Thomas, *Eccleshill;* Wm. Godward, *Free Writing School, Sheffield;* Robert Loraine, *Hexham Grammar School;* John Botterill and H. Metcalf, *R. L. School, Beverley;* Christopher Macann, *Lutton, Lincolnshire;* Charles Saunders, *Barningham, Mr. Newby's Academy, near Greta-Bridge;* John Williams, *Mr. D. T. Sheridan's Academy, Stafford.*

Master B. C. Goodison, *Rev. H. Davis' Seminary, Eaton Bishop,* answered 1, 2, 3, 4, 5, 6, 8.

Master John Godward, *Free Writing School, Sheffield,* answered 1, 3, 6, 8.

Master Wm. Vint, *Idle,* answered 1, 4, 6.

Master John Algor, *Sheffield,* 1, 6.

Masters Wm. Dyson and Frederick Fleck, *Rotherham,* answered 1, 2, 6.

To the Editor of the Leeds Correspondent.

SIR,—At page 36 of the Ladies' Diary for 1817, a remark is made by Mr. B. Biddle, asserting that the first solution to Quest. 37, No. 1, Vol. 2 of the Leeds Correspondent, is incorrect, on account of some mistake in taking or reducing the fluxion. Mr. H. Atkinson is said to have made a similar remark. Now, as I was the Editor of the Correspondent when that solution was published, I think it my duty to shew (by your permission) that it is Mr. R.'s Remark which is incorrect, and that there is no mistake, either in taking or reducing the fluxion.

In the solution alluded to, the expression to be made a maximum, is $b(c^2x - 2cx^2 + x^3) \div (b^2 + x^2)$, by making its fluxion $= 0$, and dividing by $\frac{bx}{(b^2 + x^2)^2}$, we get $x^4 + (3b^2 - c^2)x^2 - 4b^2cx + b^2c^2 = 0$, agreeing exactly (if the different notations be considered) with Mr. R's final equation; but it may be further divided by $x - c = 0$, and the result will be $x^3 + cx^2 + 3b^2x - b^2c = 0$, which is correct, and the same as that given in the solution. If Mr. R. will take the trouble to divide his equation by $x - b = 0$, he will find it reduced to the same form.

Were you disposed to retaliate on Mr. R. you might very justly find fault with him in another place of the Diary for 1817, viz. at page 42, where the solution which he has given to question 13, is manifestly erroneous. Indeed I am surprised that his solution should have found a place in a work of such celebrity. But it appears that the management of the Ladies' Diary has now fallen into hands less skilful than those of the late editor.

I know that discussions of this kind are not agreeable to you. To me they are equally painful. In justice, however, to myself, and to the work of which you are the editor, I could not but notice this rude and unmerited attack; and I hope that you will not think I have

done it improperly. I wish great and continued success to the Leeds Correspondent. It has already attained to a circulation beyond that of any other mathematical work published half-yearly or quarterly: And it is not the indulgence of a few mistaken and unfounded observations, similar to those on which I have here animadverted, that can cause the patronage of the public to withdraw itself. In a very infirm state of health, I anxiously subscribe myself,

Your well-wisher,

J. GAWTHORP,

Teacher of the Mathematics, St. James's Street,

DEC. 3rd. 1816. *Leeds.*

A new and expeditious method of proving the Square and Cube Roots.

By Mr. SAMUEL TAYLOR, *Wooley, near Wakefield.*

I. To prove the Square Root.

Rule.

1. Cast out the nines in the root and square the excess.

2. Cast out the nines in this square, the remainder (if any) after extraction, and the given number to be extracted, and call the three excesses a, b and c respectively: then if $a + b = c$, the work is supposed to be right.

Example.

```
      2268741(1506.23.
      1
    ------
25  )126
      125
    ------
3006 )18741
       18036
    ------
30122 )70500
        60244
    ------
301243)1025600
       903729
    ------
       121871
```

In the example above, the root is 1506.23 : cast out the nines and the excess is 8, whose square is 64 : then casting out the nines in this square (64), the remainder (121871) and the given number (2268741) and the three excesses are $1 = a$, $2 = b$, and $3 = c$: so that $a + b = c$, as it ought to be.

II. To prove the Cube Root.

Rule.

1. Cast out the nines in the Root and cube the excess.

2. Cast out the nines in this cube, the remainder (if any) after extraction, and the given number to be extracted, and call the excesses a, b and c, respectively: then if $a + b = c$, the work is supposed to be right.

Example (9th Bonnycastle's Arithmetic.)

$$
\begin{array}{r}
13.6666(2.3908 \\
8 \\
\hline
1389)5666 \\
4167 \\
\hline
154991)1499600 \\
1484919 \\
\hline
17142036\dot{6}4)14681000000 \\
13713629312 \\
\hline
967370688
\end{array}
$$

In this example the root is 2.3908, and the excess $= 4$, whose cube is 64 : then casting out the nines in 64, the remainder 967370688, and the given number 13.6666, and we have $a = 1$, $b = 0$, and $c = 1$ ∴ therefore $a + b = c$, as it ought.

It would be unnecessary to multiply examples as the application of the Rule must now be extremely easy to any example whatever.

ANSWERS

TO THE

MATHEMATICAL QUESTIONS

Proposed in the second Number of the second Volume.

(123) *By* Mr. JOHN BAINES, Jun.

Find two rational fractions, such, that if either of them be added to the cube root of the other, the sum in each case shall be the same.

Answered by Mr. ELIAS WEBSTER, *Armley Mills.*

Let $\left(\frac{x+y}{y}\right)^3$ and $\left(\frac{x+y}{y}.n\right)^3$ represent the two fractions; then by the question $\frac{x+y}{y} + \left(\frac{x+y}{y}.n\right)^3 = \frac{x+y}{y}.n + \left(\frac{x+y}{y}\right)^3$; whence $(n^3-1).\left(\frac{x+y}{y}\right)^2 = n-1$, or $\frac{x+y}{y} = \left(\frac{1}{n^2+n+1}\right)^{\frac{1}{2}}$. Now in order to have rational numbers, n^2+n+1 must be a square which equate with $n^2 - 2an + a^2$, then $n = \frac{a^2-1}{2a+1}$, where a may be taken at pleasure, provided it be greater than 1. If $a = 2$, we obtain $\frac{15}{35}$ and $\frac{25}{35}$ or $\frac{3}{7}$ and $\frac{5}{7}$, for the roots of two numbers that answer the question.

The same otherwise by Mr. SAMUEL JOHNSON *and* Mr. JAMES WHITEHEAD, *at Mr. J. Garside's Academy, Mixfield.*

Put x^3 and y^3 for the required fractions, then by the question $x^3 + y = y^3 + x$, or $x^3 - y^3 = x - y$, or dividing by $x - y$, $x^2 + xy + y^2 = 1$. Assume

$y = rx - 1$, then by substitution $x^2(r^2+r+1) - x(2r+1) + 1 = 1$; hence $x = \frac{2r+1}{r^2+r+1}$, and $y = rx - 1 = \frac{r^2-1}{r^2+r+1}$, where r is indeterminate. Take $r = 2$, then $x = \frac{5}{7}$ and $y = \frac{3}{7}$ the roots required.

Otherwise by Mr. JOHN PIPE.

Let the supposed values of the two fractions be z^3 and y^3. Then $z + y^3 = z^3 + y$, or $z - y = z^3 - y^3$, or $1 = z^2 + zy + y^2$. Put $rz = y$. Then $1 = z^2 + rz^2 + r^2z^2$, or $z^2 = \frac{1}{1+r+r^2}$; therefore $1 + r + r^2$ must be a square. Assume its side $= s - r$; then $1 + r + r^2 = s^2 - 2sr + r^2$, or $1 + r = s^2 - 2sr$, and $r = \frac{s^2-1}{2s+1}$ where s may be any whole number greater than unity. If $s = 2$, then $r = \frac{3}{5}$, $s - r = \frac{7}{5}$, $(s^2 - r)^2 = 1 + r + r^2 = \frac{49}{25}$, and $z^2 = \frac{25}{49}$; therefore $z = \frac{5}{7}$ and $y = rz = \frac{3}{7} \times \frac{5}{7} = \frac{15}{49}$. Consequently $z^3 = \frac{125}{343}$ and $y^3 = \frac{3375}{343}$ the fractions required.

Ingenious answers were also given by Messrs. Ambulator, Arithmeticus, Baines, Crowther, Davis, Garside, Hill, Lamplugh, Maffet, Shephard, S. H. F. and Stephenson.

(124) *By* Mr. C. HOLT, *Heath Charnock, Lancashire.*

What principal x^3 to continue x^2 years at x per cent. per ann. compound interest, will amount to $\sqrt[x]{1000x^2}$?

Answered by Mr. SAMUEL JOHNSON *and* AMBULATOR.

By substituting x^3, x^2, x and $\overline{1000x^2}^{\frac{1}{x}}$, respectively in the equation $pr^t = a$, for p, the principal, t, the time, r, the amount of 1£. for 1 year, and a the amount for the given time, we obtain $x^3 \times \left(1 + \frac{x}{100}\right)^{x^2} =$

$(1000x^2)^{\frac{1}{x}}$. Involve each side of this equation to the x^{th} power, and there results $x^{3x} \times \left(1 + \frac{x}{100}\right)^{x^3} = 1000x^2$. And by the nature of Logarithms, Log. $x \times 3x$ + Log. $\left(1 + \frac{x}{100}\right) \times x^3 = 3$ + Log. $x \times 2$. Hence by trial and error $x = 2.7534$ the rate per cent. and therefore $x^3 = 20.875$ the principal required.

The same by Mr. JOSHUA GARSIDE, *Mirfield.*

By the nature of compound interest $x^3 \times \left(1 + \frac{x}{100}\right)^{x^2} = \sqrt{1000\,x^2}$, or by involution $x^3 \times \left(1 + \frac{x}{100}\right)^{x^3} = 1000\,x^2$, and by division $x^{3x-2} \times \left(1 + \frac{x}{100}\right)^{x^3} = 1000$, and by the nature of Logarithms, Log. $x \times (3x - 2)$ + Log. $\left(1 + \frac{x}{100}\right) \times x^3$ = Log. $1000 = 3$, whence $x = 2.753436$; and $x^3 = 20.8749289$£. the principal.

Otherwise by Mr. J. CROWTHER, *Woodhouse Grove.*

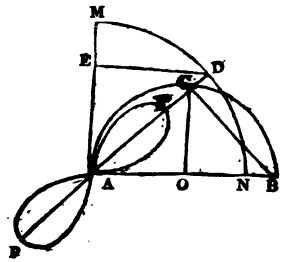
If p = principal, t = time, r = amount of 1£. for 1 year, and m = amount of p at compound interest for the time t, then it is well known $pr^t = m$, which expression by substituting the terms given in the question becomes $x^3 \times \left(\frac{100 + x}{100}\right)^{x^2} = \sqrt[x]{1000x^2}$, or, using logarithms, $\overline{3 \times \text{log. } x}$ + log. $\frac{100 + x}{100} \times x^2 = \frac{\text{log. } 1000 + \overline{\text{log. } x \times 2}}{x}$, whence by the method of trial and error we find $x = 2.7534$, and consequently $x^3 = \overline{2.7534}^3 = 20.87483$£. the sum required.

Thus nearly is the answer given by Messrs. Arithmeticus, Hill, Maffett, Shephard, Stephenson, Webster, and Whitehead.

(125) *By* Mr. A Cook, *Wooley, near Heaham.*

Let AB be the diameter of a given semi-circle, and in any chord AC take AP every where equal to CO the corresponding right sine. Required the quadrature of the curve which is the *locus* of P.

Answered by Amicus.



Let F denote the area of the circular segment AC, and f the contemporary area of the curve AP: then by the nature of the circle considered as a spiral $\dot{F} = -\frac{AC \times \dot{BC}}{2}$, and by the generation of spirals and similar triangles $AC^2 : AP^2 = CO^2 :: AB^2 : BC^2 :: \dot{F} = -\frac{AC \times \dot{BC}}{2} : \dot{f} = -\frac{BC \cdot AC \cdot BC \cdot \dot{BC}}{2AB^2}$. But by similar triangles $BC \cdot AC = AB \cdot CO$, and $BC^2 = AB \cdot BO$; take the fluxions of this equation and divide by 2, then $BC \cdot \dot{BC} = \frac{AB \cdot \dot{BO}}{2} = -\frac{AB \cdot \dot{AO}}{2}$; whence by substitution $\dot{f} = \frac{CO \cdot \dot{AO}}{4} = \frac{1}{4}$ of the fluxion of the circular segment ACO; consequently $f = \frac{1}{4}$ of the area of the segment ACO, and the whole area of the oval APA is $= \frac{1}{4} \times$ semicircular area ACB. But since the point P may be taken in AC produced, as well as in AC, the whole curve generated by that point, whilst AC from nothing becomes equal to AB, will consist of two similar and equal ovals joined together at A (which is a *punctum duplex*) resembling the curve called the *Lemniscata* or the numerical character 8, its axis PAP produced passing through the vertex C, and being equal to AB the

diameter of the semicircle, as is obvious from the genesis of the curve. Consequently the whole area of the curve = quadrant ACO.

The same otherwise by Mr. Cook, *the proposer, and* Mr. Samuel Johnson, *whose solutions were alike.*

With any radius AM (see the preceding figure) describe the quadrant MDN. Put AB = a, AM = AD = b, ED = x; then AE = $\sqrt{b^2 - x^2}$, and by similar triangles AD = b : ED = x :: AB = a : AC = $\frac{ax}{b}$ :: AC = $\frac{ax}{b}$: AO = $\frac{ax^2}{b^2}$: then $CO^2 = AP^2 = \frac{a^2x^2}{b^4} \times (b^2 - x^2)$. Now the fluxion of the arch MD = $\frac{b\dot{x}}{\sqrt{b^2 - x^2}}$ and by the Writers on Fluxions the fluxion of the area described by AP is = $\frac{AP^2 \times \text{flux. arch. MD}}{2AD} = \frac{a^2x^2}{b^4} \cdot (b^2 - x^2) \times \frac{\dot{x}}{2\sqrt{b^2 - x^2}} = \frac{a^2}{2b^4} \times \left(\frac{b^2x^2\dot{x}}{\sqrt{b^2 - x^2}} - \frac{x^4\dot{x}}{\sqrt{b^2 - x^2}} \right)$ the fluents of which are = $\frac{a^2}{2b^4} \times (\frac{1}{4} x^3 \sqrt{b^2 - x^2} + \frac{1}{8} b^2 (-\frac{1}{2}x \sqrt{b^2 - x^2} + \frac{1}{2}b \times$ (circ. arc. to rad. b and sine x); when $x = 0$, this expression vanishes, and therefore needs no correction; and when $x = b$, it gives the whole area = $\frac{a}{2b^4} \times \frac{1}{8} b^3 \times$ quadrantal arc to rad. b = $\frac{a^2}{8} \times .7854$. Therefore the area of the oval is one fourth of the semicircle ACB.

Again by Mr. J. Crowther.

With radius AM = 1, describe the quadrant MDN, and produce AC to meet it in D. Let AB = $2a$, AP = y, sine DN = x, and arc DN = w. Then (Euc. 20. 3.) the degrees in the arc BC = nNot of degrees in

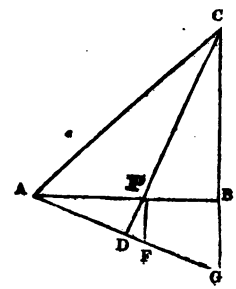
DN × 2; hence OC = a × sine of twice DN = $2ax\sqrt{1-x^2}$ = AP = y. But by the writers on fluxions (see Dealtry's Treatise, Art. Spirals) $\frac{y^2\dot{w}}{2}$ = fluxion of the area AP, which expression in the present case becomes $\frac{4a^2x^2(1-x^2)}{2} \times \frac{\dot{x}}{\sqrt{1-x^2}} = 2a^2x^2\dot{x}\sqrt{1-x^2}$; the fluent is $2a^2 \times$ ($\frac{1}{8}$ circ. zone to rad. 1 and abscissa (from the centre) $x - \frac{1}{4}x.(1-x^2)^{\frac{5}{2}}$), which needs no correction, because when $x = 0$, the expression vanishes; when $x =$ radius, then the area becomes $= 2a^2 \times$ ($\frac{1}{8}$ of semicircle to radius 1) $= 2a^2 \times .7854 = \frac{1}{4}$ of the given semicircle.

Other answers were sent by Messrs. Baines and Maffett.

(126) *By* Mr. JOSEPH SHENSTONE, *Plymouth.*

In a right angled triangle having given the greater segment of the base made by the line bisecting the vertical angle and the ratio of the base to the difference of the hypothenuse and perpendicular, to construct it.

The Analysis by Mr. ROBERT MAFFETT, *Willow-Street, Plymouth.*



Suppose it done, and that ABC is the required triangle, CP the line bisecting the vertical angle C. Produce CB to G till CG = CA; then is BG the diff. between AC and BC. Join AG and produce CP to meet it in D; and draw PF perpendicular to AB meeting AG in F; then by similar triangles AB : BG :: AP : PF; but AP is given, and the ratio of AB to BG is given; therefore PF is given ∴ AF is given; and

because in the triangles ADC, GCD, the ∠ ACD = GCD, AC = GC, and DC is common to each, they are identical or equal in all respects; therefore AD = DG, and DC is perpendicular to AG and therefore given in position; hence the construction is manifest.

The Composition by MOSES ELBEAN, Esq. *Plymouth, and* S. H. F. *of Armley.*

CON. Perpendicular to the given segment AP draw PF so that AP shall have to PF the given ratio of the base to the difference of the hypothenuse and perpendicular. Join AF, and perpendicular to it draw PD, and in AF produced take DG = DA; draw GC parallel to FP meeting AP and DP produced in B and C; join AC; then ABC will be the required triangle.

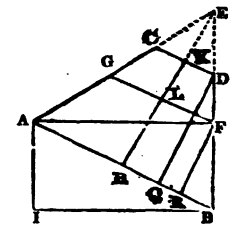
DEM. Since DC bisects AG at right angles in D, the triangles DCA, DCG are equal in all respects; therefore AC = GC, and ∠ ACP = BCP; also because CB is parallel to PF it is perpendicular to AB (by const.) and the triangles APF, ABG are similar; therefore AP : PF :: AB : BG = AC — BC, i.e. in the given ratio also AP is the given segment by construction. Q.E.D.

True and ingenious geometrical solutions are also given by Messrs. Ambulator, Arithmeticus, Crowther, Hill, Jarratt, Johnson, Lamplugh, Shenstone, Shephard, Webster, Whitehead, and algebraical ones by Messrs. Baines and Garside.

(127) *By* Mr. J. WADSWORTH, *at the Rev. Mr. Davis' Seminary, Eaton Bishop.*

A vessel in the form of a conical frustum whose base diameter is 12, top diameter 6, and depth 10 inches, is elevated till the slant side becomes perpendicular to the horizon. How much liquor will just cover the bottom when in that position?

Answered by Messrs. S. H. F. M. MACANN, *Lutton, Lincolnshire,* PYRODES, J. SHENSTONE, *and* CHARLES DE WEIGHT, *Master of the Free School, Parsondrove, near Wisbeach.*



Let ABDC, in the marginal figure, represent the given conical frustum, AF the surface of the liquor parallel to the horizon or perpendicular to DB; draw DQ, FR perpendicular to AB. By similar triangles DB : QB :: AB : BF $= \frac{AB.QB}{DB}$ :: BF $= \frac{AB.QB}{DB}$: BR $= \frac{AB \cdot QB^2}{DB^2} = \frac{12 \times 9}{109} = \frac{108}{109}$; also BQ : DQ :: BR $= \frac{AB \cdot QB^2}{DB^2}$: FR $= \frac{AB.QB.QD}{DB^2} = \frac{12 \times 3 \times 10}{109}$ $= \frac{360}{109}$; then GF $= AB - 2BR = \frac{1092}{109}$. And by pa. 307, Simpson's Algebra, the solidity of the ungula AFB $= \left(\frac{AB^2 - GF\sqrt{AB.GF}}{AB - GF}\right) \times AB \times FR \times .2618 = 178.829$ cubic inches, the quantity of liquor required.

The same otherwise by Mr. E. WEBSTER.

'Let ABE (in the preceding figure) represent the cone completed; then as AB $\div$ CD : HK :: CD : KE $= 10$, therefore BE $= \sqrt{HE^2 + BH^2} = 20.880613$; also by similar triangles BE : BH :: AB : AI $=$ BF $=$ 3.4481746, and AF $= \sqrt{AB^2 - AI^2} = 11.49393$. Again BE : BH :: EF : LF $= 5.0091743$, and by conics $\sqrt{GF \times AB} = 10.96449$ the conjugate axe of the ellipse $= C$; consequently AF $\times$ C $\times$ EF $\times$ .2618 $=$ 575.15482 $=$ solidity of AFE, which being subtracted from 753.984, the solidity of the whole cone, leaves 178.8292 cubic inches, the required quantity.

Ingenious solutions to this question were also received from Messrs. Ambulator, Arithmeticus, Anonymous, Baines, T. Charlton, Crowther, Elbean, Garside, Johnson, Lamplugh, Maffett, J. Pipe, Sampson, Shephard, Wadsworth, and Whitehead.

(128) *By* Mr. R. SMITH, *Manchester.*

Admit Jack to be the greatest trump that a certain person A holds at the game of "All Fours" when the trump turned up is less than Jack. It is required in this case to determine the odds against his adversary B, he holding one greater than Jack at the same deal.

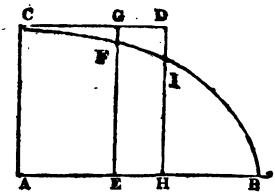
Solution by the Proposer.

It is clear that when the six cards that A holds and the trump turned up are taken from the whole pack, viz. 52, there remain 45, among which are three trumps greater than Jack, viz. *Ace, King, and Queen,* which being again subtracted leave a remainder of 42, which may be considered as blanks. Now for A's Jack to be the highest trump under the conditions of the question, B must hold six blanks, the probability of which (Cor. 2, Prob. 6. Simpson's Laws of Chance,) is $\frac{42.41.40.39.38.37}{45.44.43.42.41.40}$ $= \frac{9139}{14190}$ = the probability that B does not hold a greater trump than Jack; therefore $1 - \frac{9139}{14190} = \frac{5051}{14190}$ = the probability that B does hold a greater trump than Jack; consequently the odds are 9139 to 5051 or as 9 to 5 nearly against B.

(129) *By* Mr. W. WISEMAN, *Hull.*

A cask of the first variety being a figure formed by the revolution of the middle semi-segment of an ellipse about its transverse axis, it is required to determine the difference in wine gallons between this solid and the one generated by the revolution of the same segment about a tangent to the curve at the extremity of the conjugate semi-axis: the length of the segment, the semi-conjugate, and the end ordinate being 34.6, 16.2 and 13.1 inches respectively.

Solution by the Proposer.



Let AB $= t = 29.408$, be the semi-transverse, AC $= c = 16.2$ the semi-conjugate diam. AH $= l =$ 17.3 the semi-length, HI $= 13.1$, AE $= x$, GF $= y$, and $p = 3.1416$. Then per property of the ellipse $t^2 : c^2 :: t + x \times t - x : \mathrm{EF}^2 = \frac{c^2}{t^2} \times (t^2 - x^2)$, whence $\mathrm{GF} = y = c - \frac{c}{t}\sqrt{t^2 - x^2}$ and $py^2\dot{x} = \dot{S} = p\dot{x} \times (c - \frac{c}{t}\sqrt{t^2 - x^2})^2 = \frac{pc^2}{t_2} \times (2t^2 \dot{x} - 2t\dot{x}\sqrt{t^2 - x^2} - x^2\dot{x}$, wherefore $S = \frac{pc^2}{t^2} \times (2t^2 x - \frac{1}{3}x^3 - 2t \times$ (semi-circular area to rad. t and cos. x,)) or (when $x = l$) $= pc^2 l \times \left(2 - \frac{l^2}{3t^2} - \frac{2a}{tl}\right)$ which together with $pc^2 l \times \left(1 - \frac{l^2}{3t^2}\right)$, the content of the figure first mentioned in the question, being taken from the cylinder $pc^2 l$, leaves $2pc^2 l \times \left(\frac{l^2}{6t^2} - 1 + \frac{a}{tl}\right)$ for half the required difference; therefore $\frac{4pc^2 l}{231} \times \left(\frac{l^2}{3t^2} - 1 + \frac{a}{tl}\right)$ $= 13.391$ wine gallons is the difference required.

Ingenious solutions were also received from Messrs. Crowther, Garside, Johnson, Maffett, and Webster.

(130) *By* Mr. J. CROWTHER, *Woodhouse Grove.*

If from any number whatever the sum of the digits composing it be subtracted, the remainder shall be a multiple of another certain number, which it is required to determine by an algebraic investigation.

Answered by Messrs. WILLIAM SHEPHARD, *Eccleshill, and* J. WHITEHEAD, *Mirfield.*

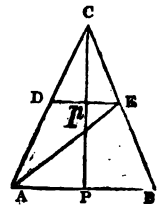
Let the digits of any number be a, b, c, d, &c. then the number will be expressed by $a+10b+100c+1000d$ &c. and the sum of its digits by $a+b+c+d$ &c. the difference of these is $9b+99c+999d$ &c. which is always a multiple of 9; therefore 9 is the number required.

And exactly in this manner is the solution given by our ingenious correspondents Messrs. Ambulator, Arithmeticus, Baines, Crowther, Cook, Davies, Garside, Jarratt, Johnson, J. Snibe, Stephenson, and Webster.

(131) *By* W.

If a right cone be divided into any two parts by a plane passing through the edge of its base, and from the vertex of the section another plane be drawn parallel to the base: the top part cut off by the first plane shall be a mean proportional between the top part cut off by the second plane and the whole cone. Required a proof.

Answered by Mr. J. GARSIDE, *and* Mr. J. HILL, *Kilham Academy.*



Let ACB be a vertical section of the cone through its axis, AE the section passing through A, DE that which is parallel to AB, and CP the cone's altitude. Put AB $=$ D, DE $= d$, PC $=$ H, and $n = .7854$; then by similar triangles $Cp = \frac{dH}{D}$. Now the whole cone ACB $= \frac{1}{3}nHD^2$, the top part DEC $= \frac{nHd^3}{3D}$, and the oblique cone CAE $= \frac{1}{3}nHd\sqrt{Dd}$ (see cor. 4, p. 170, Dr. Hutton's Mens. 4th edit.). But $\frac{1}{3}nHd\sqrt{Dd} = \sqrt{\frac{1}{3}nHD^2 \times \frac{\frac{1}{3}nHd^3}{D}}$ Q. E. D.

The same otherwise by Messrs MAFFETT *and* WEBSTER.

By similar cones, cone ACB : cone DEC :: AB^3 : DE^3 :: (see the cor. cited above) the square of the cone ACB : square of the oblique cone AEC; whence alternately the square of ACB : ACB :: the square of AEC : DEC :: ACB : 1, therefore ACB × DEC = square of AEC, or AEC is a mean proportional between ACB and DEC. Q.E.D. *Nearly thus is the Demonstration given by Mr. Crowther.*

This curious property was also demonstrated as in the first solution above by Messrs. Baines, Cook, Johnson, and Whitehead.

(132) *By* Mr. J. GARSIDE, *Mirfield.*

Having lately to survey a beautiful plantation in form of a triangle, I observed that one of its sides bore directly N.E. by N. and another S.E. by E. and the bearing of the third side was 3° 7′ 11½″ from the East towards E. by N. Required the sides and area of the plantation, its perimeter being 300 chains.

Answered by Messrs. J. BAINES, T. CHARLTON, T. LAMPLUGH, *and* R. WEST.

From the data the angles of the plantation are readily found to be 90°, 53° 7′ 48½″ and 36° 52′ 11½″, the natural sines of which viz. 1, and, 8, and, 6 respectively, being proportional to their opposite sides, it will be as (1. + .8 + .6) = 2.4 : perim. 300 :: 1, and .8 and .6 : to 125, and 100, and 75 the sides of the plantation; then $\frac{100 \times 75}{2}$ = 3750 sq. chains = 375 acres, the area required.

Otherwise by PYRODES.

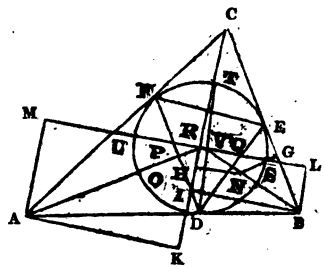
In this question are given the ∠ A = 53° 7′ 48½″, ∠ C = 36° 52′ 11½″, ∠ B = 90°, and the perimeter = 300 to find the sides and area of the plantation. First as cot. ½A : tan. ½C :: 150 = half the perim. : 150 — AC = 25 : hence ½AC = 125, then by Trigonometry we find AB = 75 ∴ BC = 300 — 125 — 75 = 100; then $\frac{BC \times AB}{2}$ = 3750 chains, or 375 acres the area required.

Other ingenious answers were given by Messrs. Ambulator, Anonymous, Crowther, Elbean, Garside, Johnson, Macann, Maffett, Pipe, Shenstone, Webster, and C. D. Weight.

(138) *By* Mr. S. JOHNSON.

Given the diameter of the inscribed circle and the radii of the circles inscribed in the two triangles made by the line, bisecting the vertical angle and terminating in the base of a plane triangle, to construct it.

Answered by Mr. J. H. SWALE, *Brunswick Place, Liverpool.*



Suppose the △ ACB constructed with the proposed data. To R, the centre of the inscribed circle, draw the lines AR, BR, CR, the two former passing through O, N, the centres of the other two inscribed circles; draw RE, NG, to E, G, two of the points of contact, and let AK, BI, NH be perpendicular to CR. Because of parallels RB : BN :: RE : NG, and of course RB : RN :: RE : RE — NG :: BI : NH :: BI : NG are given ratios, or BI a given line. In the same way we shew that AK is a given line, and the prob. is, to construct the △ having given the radius of the inscribed circle, and the two perpendiculars from the angles at the base upon the line bisecting the vertical ∠. We proceed to consider the problem in its new form. Draw MRL parallel to AK or BI meeting the circle in U, S, and AM, BL parallels to CK, in M, L. Make SQ : QU :: LS : LU, UP : SP :: MU : MS, then MR = AK, LR = BI, and of course ML = AK + BL, being given lines, it follows that Q, P, are given points in the diameter SU; and it is well known to Geometers (see the solutions of Quest. 75, Boston Enquirer, or

those of the Prize Question 240, Mathematical Repository, Old Series) that, if, from any points A, B, in the perpendiculars MA, LB tangents to be drawn to the circle, the lines joining the points of contact will pass through the points P, Q. Hence, if D, E, F be the points of contact, DE, DF will pass through Q, P and EF will be perp. to CK or parallel to SU. Let CK cut the circle in T and join DT meeting PQ in V. By reason of pars. DQ : QE :: DP : PF. or DQ . QE : DQ^2 :: DP . PF : DP^2 : alternately DQ^2 : DP^2 :: DQ . QE : DP . PF :: SQ QU : UP . PS = a given ratio, or DQ : DP a given ratio; that is, as DT obviously bisects the angle EDF, the ratio QV : VP, and consequently the points V, D, are given, and the inscription of the △ DEF obvious, from which the △ ACB is immediately determined. This is a particular case of the Prob. To inscribe a △ in a given circle, two of whose sides shall pass through two given points, and the remaining side be parallel to a line given in position.

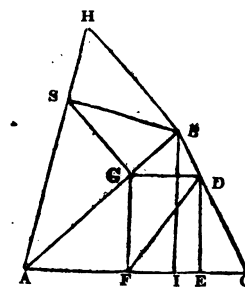
It was also answered geometrically by Messrs. Crowther, and Johnson.

(134) *By the same gentleman.*

In a given plane triangle to inscribe a rectangle whose diagonals shall be the least possible.

A geometrical solution by Mr. J. H. SWALE, *and* Mr. WM. WRIGHT, *Master of the Mathematical School, Hull.*

CONST. Let ABC be the given triangle. Draw BI, BH perpendicular to AC, AB, and take BH a fourth proportional to AC, AB, and BI : on AH joined demit the perpendicular BS, and on AB the perpendicular SG; then will G be one of the angular points of the required rectangle GDEF.



DEM. Draw the diagonal DF. By construction and similar triangles AC : AB :: BI : BH :: GF : GS :: GD

: GB, wherefore because the triangles GDF, GBS, have the sides about the right angles FGD, SGB proportionals, they are similar, therefore GD : GB :: DF : BS :: AC : AB = a given ratio; but since BS is perpendicular to AH it is the least line that can be drawn from B to terminate in AH; consequently DF which has a given ratio to BS is the least possible. Q.E.D.

Elegant geometrical constructions were also given by Messrs. J. Crowther and S. Johnson.

A fluxional solution by AMBULATOR, Mr. J. GARSIDE, *and* Mr. T. LAMPLUGH.

Let BI = a, AC = b, and GF = x, then by similar $\triangle$s BI : AC :: BI — GF : GD = $\frac{b}{a} . (a - x)$, whence (by Euc. 47, 1) $FD^2 = x^2 + \frac{b^2}{a^2} . (a - x)^2 =$ a *min.* per question, or $a^2x^2 + b^2 . (a^2 - 2ax + x^2)$ is a *min.* the fluxion of which being put = 0, and reduced, gives $x = \frac{ab^2}{a^2 + b^2}$ = GF, then GD = $\frac{b}{a} . (a - x)$ = $\frac{ba^2}{a^2 + b^2}$ the dimensions of the rectangle.

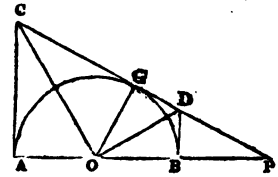
Other solutions were also given by Messrs. Baines, Macann, and Charles De Weight.

The next eight questions are selected from the *Cambridge Senate House Problems*, given to the Candidates for honors during the examinations for the degree of B.A. in January 1814 and 1815.

(135) If from the extremities of the diameter of a circle tangents be drawn and produced to intersect a tangent to any point in the circumference, the right lines joining the points of intersection and the centre of the circle shall form a right angle.

Solution by AMBULATOR, *and* Mr. J. SHENSTONE.

Let AB be the diameter, O the centre of any circle, AC, BD tangents at A, B; CGD a third tangent at G, meeting AC, BD at C, D; join OC, OD. The $\triangle$s ACO,



GCO are mutually equal, having $\angle AOC = \angle COC$; also the $\triangle$s BOD, GOD, have $\angle BOD = \angle GOD$; or the $\angle$s AOC and BOD together equal the angle COD, and therefore each equal a right angle, the three angles together being two right $\angle$s.

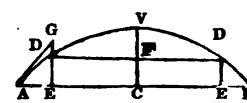
Mr. Swale, after giving the demonstration, as above, adds the following one. Join OG, and let AB, CD meet at P. Because of similar triangles PB : BD :: PG : GO :: PA : AC, and PA . PB : AC . BD :: $PG^2 : GO^2$ :: PA . PB : CG . GD; but PA . PB = PG^2, hence CG . GD = GO^2, which is a property of a $\triangle$ right angled at O, This property holds true in the other Conic Sections taking O the focus of the section and AB the transverse.

Ingenious demonstrations were also received from Messrs. Baines, Cook, Crowther, Elbean, S. H. F. Garside, Johnson, Lamplugh, Maffett, Pipe, Pyrodes, Webster, Whitehead, Wiseman, and W. Wright.

(136) Given the greatest range of a projectile upon an horizontal plane: determine at what distance from the point of projection an object whose perpendicular height is (*d*), must be situated, so that the projectile may just strike the top of it.

Answered by Messrs. S. JOHNSON, *and* E. WEBSTER.

Let AB be the given range, DE = *d*, the perp. height of the object, ADB the parabola described by the projectile, and AG a tangent at A meeting ED produced in G. Since AB is the greatest range the $\angle$ GAE = 45° (cor. 5, pa. 155, vol. 2, Hutton's Course, sixth. ed.) therefore AE = GE; but (theor. 14, Hutton on the parabola,) DG : DE :: AE : BE, componendo GE = AE : DE :: AB : BE: therefore AB × DE = AE × BE = a given rectangle, or the sum and rectangle of two lines are given to determine them; the method of doing which is well known.



The same otherwise by Messrs. COOK *and* WISEMAN.

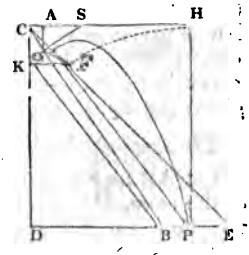
Let AB be the given range, D the place of the object, DE perpendicular and DD || to AB, V the ver-

tex, and VC the axis of the parabola described by the projectile. Then AB (being the greatest range) will be the parameter (by the cor. quoted above) also VC = $\frac{1}{4}$AB. Put AB = p, then $p \times \frac{1}{4}p - d = FD^2 = CE^2$, or CE = $\sqrt{\frac{1}{4}p^2 - pd}$; whence AC $\pm$ CE = AE, or AE = $\frac{1}{2}p \pm \sqrt{\frac{1}{4}p^2 - pd}$, the distance required.

Nearly thus is the solution given by Messrs. Ambulator, Anonymous, Baines, T. Charlton, Crowther, Elbean, Garside, Maffett, Pipe, Pyrodes, and Shenstone.

(137) A hollow cone whose vertical angle is 60°, is filled with water, and placed with its base downwards. It is required to determine the place where a small orifice must be made in its side, so that the issuing fluid may strike the horizontal plane in a point, the distance of which from the bottom of the vessel is to the distance of the orifice from the top as 5 to 4.

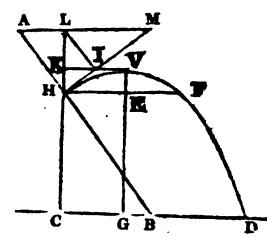
A geometrical solution by AMICUS.



Analysis, on the hypothesis that water issues from a jet with the velocity acquired by a body in falling through the whole perpendicular height of the incumbent column. Let CB be the side of the cone given in position and magnitude, O the centre of the required hole in CB, and P the point where the issuing fluid strikes the horizontal plane. Draw CH parallel and PH, OA perpendicular to the horizon, also draw OS perp. to CB; then because the velocity of the water in OS = that acquired in falling through AO (by hypothesis) CH is the directrix of the parabola passing through the points O, P, and the angle AOC is given = 30° (per quest.) therefore the angle AOS = ACO is given = 60°. Draw FO = AO making the angle FOS = AOS, then F is the focus (by conics) and the angle FOC is given = 150°. Draw FK parallel and equal to the horizontal distance PB, and let CF, CK joined meet PB

in E and D, also join PF, BK. Because the angles of the triangle AOC are given, the triangle is given in species (43 Euclid's data); therefore the ratio of CO to AO or FO is given; wherefore because the angle COF is given, the triangle FCO is given in species (44 data); therefore the angles OCF, OFC are given, and since the angle OCA is given, FCH = FED is a given angle; therefore (35 dat.) CE is given in magnitude. Now by the question the ratio of PB or FK to CO is given, and because the triangle FCO is given in species, the ratio of CF to CO is given; therefore (dat. 9) CF has to FK a given ratio; therefore (Euc. 4, 6) CE has to ED a given ratio, and because CE is given, ED is given in magnitude (dat. 2) and it is also given in position, and the point E is given, wherefore the point D is given (dat. 30), therefore (dat. 29) CD is given in position and magnitude. But by conics PF = PH, and (Euc. 33, 1) BK = PF; therefore BK = PH a given line, and the point B is given, therefore the point K is given (28 dat.) and the angle CKF is given, therefore (32 dat.) FK is given in position, and the point F is given (28 dat.) wherefore because the angle OFC is given, FO is given in position (32 dat.) therefore the point O is given (28 dat.). Ergo solutum.

The same algebraically by Mr. W. WISEMAN, *Hull.*



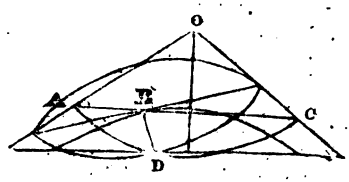
Let AB be the slant side of the cone, H the centre of the hole in AB required HVFD the parabola described by the issuing water, HM a tangent at H, VG the axis, CGBD the horizon, HEF a double ordinate cutting VG in E, CL drawn through H perpendicular to CD meeting AL a parallel to CD in L, and LI, IK perpendicular to HM, HL. By the question the angle AHL (= CHB = HLI = HIK,) being = 30°, therefore AH = 2AL, HL = 2HI, HI = 2HK,

or HL $=$ 4HK; but by the theory of projectiles HK is $=$ VE, and 4IK $=$ HF; therefore putting AH $= x$, and AB $= a$, we have HL or 4 HK $= \frac{x}{2}\sqrt{3}$, HK or VE $= \frac{x}{8}\sqrt{3}$, and IK $= \frac{3x}{8}$, or EF $= \frac{3x}{4}$; then by the parabola VE : EF^2 :: VG : GD^2, or $\frac{x}{8}\sqrt{3} : \frac{9}{16}x^2 :: \frac{1}{2}\sqrt{3} \times \sqrt{a-x} + \frac{x}{8}\sqrt{3} : \frac{9}{16}x \times \sqrt{4a-3x} = GD^2$, or GD $= \frac{3}{4}\sqrt{4ax - 3x^2}$, also BC $= \frac{1}{2}$ BH $= \frac{1}{2}a - \frac{1}{2}x$, and BG $= (\frac{1}{2}a - \frac{1}{2}x) - \frac{3}{4}x = \frac{1}{2}a - \frac{5}{4}x$. Per question BD $= \frac{5}{4}x - \frac{1}{2}a + \frac{3}{4}\sqrt{4ax - 3x^2}$: AH $= x :: 5 : 4$, whence multiplying means and extremes, &c. we get $9x \times (4a - 3x) = 4a^2$, and lastly $x = \frac{2a}{3} \times (1 \pm \frac{1}{3}\sqrt{6})$.

We regret that our limits will not admit of the ingenious solution by Mr. Cook, it being on the same principle as those above. Solutions were also given by Messrs. Ambulator, Anonymous, Garside, and Maffett.

(138) If an hyperbola and its asymptotes revolve about the axis major, prove that all the sections of the cone made by planes touching the hyperboloid have the same axis minor, which is the axis minor of the hyperbola.

Solution by Mr. Cook, *and* Mr. J. Crowther, *Woodhouse Grove.*



Let AC be drawn parallel to the conjugate axis cutting the asymptotes AO CO, in A, C, and the hyperbola in B; then per conics the semi-axis minor of the hyperbola is a mean proportional between the segments AB and BC. Let a section of the cone touch the

hyperboloidin B, then B will be the centre of the ellipse (by conics); draw BD the semi-axis minor of the ellipse, and it will be an ordinate to the circle ADC, which is a section of the cone perpendicular to its axis, and since, by the circle, BD is a mean proportional between the segments AB and BC, it follows, that all the sections of the cone made by planes touching the hyperboloid have the same axis minor which is the axis minor of the hyperbola. Q.E.D.

Nearly thus it is answered by Messrs. Johnson and Wiseman.

(139) Determine the nature of the curve in which $\frac{\dot{x}}{x} + \frac{\dot{y}}{y} : \frac{\dot{x}}{y} + \frac{\dot{y}}{x} :: n : 1$, x and y being the abscissa and ordinate.

Answered by Messrs. WEBSTER *and* WRIGHT.

By bringing the two first terms to a common denominator $\frac{y\dot{x} + x\dot{y}}{xy} : \frac{x\dot{x} + y\dot{y}}{xy} :: n : 1$, or $y\dot{x} + x\dot{y} : x\dot{x} + y\dot{y} :: n : 1$, or $y\dot{x} + x\dot{y} = n(x\dot{x} + y\dot{y})$; taking the fluents $xy = \frac{n}{2}(x^2 + y^2)$, or $x^2 - \frac{2xy}{n} = -y^2$, whence $x = y \cdot \left(\frac{1}{n} \pm \frac{1}{n}\sqrt{1 - n^2}\right)$ an equation to a straight line.

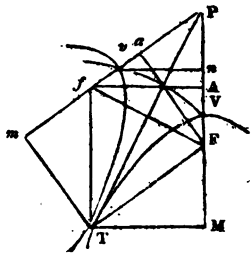
The same by Mr. JONATHAN CROWTHER.

By multiplying means and extremes, we have $n\left(\frac{\dot{x}}{y} + \frac{\dot{y}}{x}\right) = \frac{\dot{x}}{x} + \frac{\dot{y}}{y}$, or $\frac{n \cdot (x\dot{x} + y\dot{y})}{xy} = \frac{\dot{x}y + x\dot{y}}{xy}$, or $n \cdot (x\dot{x} + y\dot{y}) = \dot{x}y + x\dot{y}$; and taking the fluents $\frac{n}{2} \cdot (x^2 + y^2) = xy$, or $n \cdot (x^2 + y^2) = 2xy$; whence by reduction we obtain $x = \frac{y}{n}(1 \pm \sqrt{1 - n^2})$ an equation to a line of the first order or a straight line.

Ingenious answers were also given by Messrs. Ambulator, Cook, Gasride, Johnson, Muffett, and Wiseman.

(140) If two equal parabolas be placed in such a manner that they may touch each other at their vertices, and one be made to roll upon the other, its focus will describe a right line, and the vertex a cissoid, the diameter of whose generating circle is equal to half the latus rectum of the parabola.

Answered by Messrs. COOK *and* JOHNSON.



Let F be the focus of the fixed parabola, V its vertex, f and v those of the revolving parabola, and T the point of contact. Produce the axes FV, fv to meet in P, and join Ff, TF, Tf, and TP. Since the parabolas are mutually equal (as well as the arcs VT, vT) $\therefore$ FT $= f$T, also TP will be a common tangent, to both the curves at T; whence per conics, FT $=$ FP $= f$T $= f$P; therefore the figure PFTf is a rhombus, and Tf is $\parallel$ to the axis VF; therefore (by cor. 1, theorem 4, vol. 2, Hutton on the parab.) the point f describes the directrix of the fixed parabola.

Again, draw fA, Fa, perpendicular to PF, Pf, then because the triangle PFf is isosceles, the right angled triangles fAF, Faf are similar and equal in all respects; therefore $fa =$ FA; hence since $fv =$ FV, $av =$ AV $=$ VF $= \frac{1}{2}vf$; but $fa =$ FA is given, and the point F and Af are given in position; therefore the point a (by the definition of that curve) moves in the Cissoid of Diocles, the diameter of whose generating circle is $=$ to af, half the latus rectum of the parabola.

The same otherwise by Messrs. MAFFETT *and* WRIGHT.

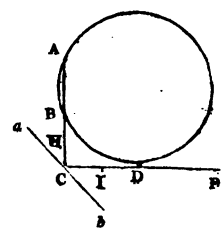
These gentlemen after demonstrating, nearly as above, that the focus of the revolving parabola moves in the directrix of the fixed one, proceed thus. Draw

TM, vn, (see preceding fig.) perpendicular to FP, and put $a = VF = VA$, $Vn = x$, $vn = y$, $VM = VP = vP = z$; then by the parabola $FT = FP = a + z$, $FM = z - a$, and $TM = 2\sqrt{az}$, and per sim. $\triangle$s, $FT : TM :: vP : vn = y = \frac{2z\sqrt{az}}{z+a}$; also $FT : FM :: vP : Pn = \frac{z^2 - az}{z+a}$. Now $Vn = VP - Pn = z - \frac{z^2+az}{z+a} = \frac{2az}{z+a} = x$; hence $z = \frac{ax}{2a-x}$, and $y = \frac{2z\sqrt{az}}{z+a} = \sqrt{\frac{x^3}{2a-x}}$ or $x^3 = y^2 \times (2a - x)$ an equation to the Cissoid, the diameter of whose generating circle is $= 2a$. (See art. Cissoid, Dr. Hutton's Dictionary.)

Similar to one or other of these are the solutions by Messrs. Jonathan Crowther, and William Wiseman.

(141) Two elastic balls beginning to descend from different points in the same vertical line, impinge on a perfectly hard plane inclined at an angle of 45°, and move along a horizontal plane with the velocities acquired. Shew that if a circle be described passing through the two points from which the balls began their motion, and touching the horizontal plane, the point of contact will bisect the distance between the vertical line and the point where they impinge on each other.

Answered by Mr. Cook, *Wooley, near Hexham.*



Let A and B be the points in the vertical AC from which the balls (A and B) begin to fall, C the point where they strike the inclined plane ab, CP their horizontal direction afterwards, and P the point of impact; also let D be the point where a circle described through A and B touches CP.

Put $16\frac{1}{12} = G$, then from the principles of Dynamics the velocity acquired by falling through $AC = 2\sqrt{G.AC}$ and

the time $= \sqrt{\frac{AC}{G}}$; the velocity acquired through BC $= 2\sqrt{G.BC}$, and the time $= \sqrt{\frac{BC}{G}}$. Then $\frac{CP}{2\sqrt{G.AC}} =$ the time of A's passing from C to P with the velocity $2\sqrt{G.AC}$: and $\frac{CP}{2\sqrt{G.BC}} =$ the time of B's passing from C to P with the vel. $2\sqrt{G.BC}$. Now the difference of the times of the two bodies passing over CP is equal to the difference of the times in falling through AC and BC; hence $\frac{CP}{2\sqrt{G.BC}} - \frac{CP}{2\sqrt{G.AC}} = \sqrt{\frac{AC}{G}} - \sqrt{\frac{BC}{G}}$, or $\frac{CP}{2\sqrt{BC}} - \frac{CP}{2\sqrt{AC}} = \sqrt{AC} - \sqrt{BC} \therefore CP = 2\sqrt{AC.BC}$; but (by Euc. 36. 3.) $AC.BC = CD^2$; consequently $CP = 2CD$. Q.E.D.

The same otherwise by Messrs. JONATHAN CROWTHER, *and* WM. WISEMAN.

Let A and B be the points from which the balls begin their motion, C the point where they meet the horizontal plane, P that of impact, D the point of contact of CD and the circle passing through A, B; H the place of the ball A when the ball B arrives at C, and I the place of the ball B when A arrives at C. Put $AC = S$, $BC = s$, $16\frac{1}{12} = g$, and put T, t and V, v for the times and velocities in AC and BC respectively: then $AH = BC$, being described in the time t, and AC in the time T, CH and CI will be described in the time $T - t$; whence $1'' : T - t :: v : v \times \overline{T - t} = IC$; also it is plain that when A sets off from C it will approach the ball B (at I) with the difference of the velocities $V - v$; whence $V - v : v \times \overline{T - t} :: V : \frac{Vv \times \overline{T - t}}{V - v} =$

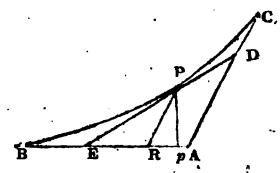
$$CP = \text{(by substitution)} \frac{2\sqrt{gS} \times 2\sqrt{gs} \times \left(\sqrt{\frac{S}{g}} - \sqrt{\frac{s}{g}}\right)}{2\sqrt{gS} - 2\sqrt{gs}}$$

$= 2\sqrt{Ss} = 2\sqrt{AB.BC} =$ (by the circle) 2CD, or the point D bisects CP. Q.E.D.

Ingenious solutions were also given by Messrs. Johnson, Maffett and Wright.

(142) The velocities of two bodies A and B are in a given ratio, and they begin to move at the same time from A and B, the extremities of a given line AB; A moving uniformly in a direction inclined at a given angle to AB, and B uniformly in the direction BA. Determine the nature of the curve to which the line joining the bodies is always a tangent.

Answered by Mr. SAMUEL JOHNSON.



Let AC be the given direction in which A moves, and C its position when B arrives at A; then per question AB, AC will be tangents to the required curve at B and C. Let E and D be any simultaneous situations of the two bodies; then because they move uniformly AC : AB :: AD : BE, alternando, AC : AD :: AB : BE, dividendo, CD : DA :: AE : EB, a property of the parabola to which AB, AC and the line joining the points E, D, are tangents. (Vide Em. Conics, Prop. 51, B. 3, or Hutton's Conics, Parab. Theo. 25.) Therefore the curve required is the Parabola.

In this manner nearly the question is answered by Messrs. Crowther and Maffett.

The same otherwise by Mr. A. COOK.

Let BA be the given line or direction in which B moves, and AC the given direction in which A moves; also let BPC be the curve and ED a tangent to it at P: draw Pp perpendicular to BA, and PR || CA. Put BA $= a$, sin. $\angle$ CAB $=$ sin. $\angle$ PRp $= s$, its cos. $= c$, Bp $= x$, Pp $= y$, and let the velocity of B : velocity

of A :: 1 : r. Then $pR = \frac{cy}{s}$, $PR = \frac{y}{s}$, $Ep = \frac{\dot{y}x}{\dot{y}}$, $ER = \frac{\dot{y}x}{\dot{y}} - \frac{cy}{s}$, $BE = x - \frac{\dot{y}x}{\dot{y}}$, $EA = a - x + \frac{\dot{y}x}{\dot{y}}$; and by similar triangles $ER : RP :: EA : AD = \frac{a\dot{y} - x\dot{y} + \dot{y}x}{s\dot{x} - c\dot{y}}$; but by the question $1 : r :: BE = x - \frac{\dot{y}x}{\dot{y}} : AD = r \times \left(\frac{\dot{y}x - y\dot{x}}{\dot{y}}\right)$: consequently $\frac{a\dot{y} - x\dot{y} + \dot{y}x}{s\dot{x} - c\dot{y}} = r \cdot \left(\frac{\dot{y}x - y\dot{x}}{\dot{y}}\right)$. By reduction $a\dot{y}^2 - (1 - cr)\, x\dot{y}^2 + (1 - cr)y\dot{x}\dot{y} = sr \times (x\dot{y}\ddot{x} - y\dot{x}^2)$, or (putting $m = 1 - cr$,) $sr \times (x\dot{y}\ddot{x} - y\dot{x}^2) = a\dot{y}^2 - mx\dot{y}^2 + myx\dot{y}$. Now to find the relation of x to y, suppose $y = Ax^2 + Bx^3 + Cx^4 + Dx^5$ &c., then $\dot{y} = 2Ax\dot{x} + 3Bx^2\dot{x} + 4Cx^3\dot{x} + 5Dx^4\dot{x}$ + &c. $\dot{y}^2 = 4A^2x^2\dot{x}^2 + 12ABx^3\dot{x}^2 + (16AC + 9B^2)\, x^4\dot{x}^2$ + &c. Whence by substituting these values in the above equation and equating the like terms we find $A = \frac{rs}{4a}$, $B = \frac{mrs}{8a^2}$, $C = \frac{5mrs}{64a^3}$, $D = \frac{7m^2rs}{128a^4}$, &c. Hence $y = rs \times : \frac{x^2}{4a} + \frac{mx^3}{8a^2} + \frac{5m^2x^4}{64a^3} + \frac{7m^3x^5}{128a^4}$, &c.; but the sum of this series is $= \frac{rs}{m} \times \left\{\frac{2a}{m}\left(1 - \sqrt{1 - \frac{mx}{a}}\right) - x\right\}$. Therefore $y = \frac{rs}{m} \times \left\{\frac{2a}{m} \cdot \left(1 - \sqrt{1 - \frac{mx}{a}}\right) - x\right\}$ the equation of the curve.

When $x = 0$, this expression gives $y = 0$, and when ED coincides with AC, or $x = a + rac$, it gives $y = ras$, as it ought.

AMICUS, adopting Mr. Cook's notation finds like him $srx\dot{y}\dot{x} - sry\dot{x}^2 = a\dot{y}^2 - mx\dot{y}^2 + myx\dot{y}$, and then determines the relation of x to y as follows. By transposition and division $(a - mx)\frac{\dot{y}^2}{\dot{x}^2} + (my - srx)\frac{\dot{y}}{\dot{x}} + sry = 0$, (1). Taking the fluxion of this equation supposing $\dot{x}$ constant, and we have $\left\{2(a - mx)\dot{y} + (my - srx).\dot{x}\right\} \times \ddot{y} = 0$, hence $(2a - 2mx)\dot{y} + (my - srx)\dot{x} = 0$, (2) and $\ddot{y} = 0$. From the equation (2) we have $\frac{\dot{y}}{\dot{x}} = \frac{srx - my}{2a - 2mx}$, and this being substituted for $\frac{\dot{y}}{\dot{x}}$ in equation (1) we obtain after due reduction $4rsy \times (a - mx) = (srx - my)^2$; hence $4asry = (srx + my)^2 = s^2r^2x^2 + 2rsmxy + m^2y^2$; and lastly $y = \frac{rs}{m^2} \times (2a - mx - 2\sqrt{a^2 - amx})$ the equation of the curve.

Mr. William Wiseman also gave an ingenious solution.

THE

LEEDS CORRESPONDENT,

A

Literary, Mathematical and Philosophical Miscellany.

Tarda fluunt ingrataque tempora, quæ spem
Consiliumque morantur agendi gnaviter id, quod
Æque pauperibus prodest, locupletibus æque;
Æque neglectum pueris senibusque nocebit.

HORAT.

No. IV. JULY, 1817. Vol. II.

Translations are respectfully requested of those, among our junior correspondents, who have not attained to sixteen years of age.

JUVENILE DEPARTMENT.

Effets de l'Amitié.

C'est l'insuffisance de notre être qui fait naître l'amitié; et c'est l'insuffisance de l'amitié qui la détruit. Est-on seul? On sent sa misère; on sent qu'on a besoin d'appui; on cherche un fauteur de ses goûts, un compagnon de ses plaisirs et de ses peines; on veut un homme dont on puisse occuper et le cœur et la pensée: alors l'amitié paroît être ce qu'il y a de plus doux au monde. A-t-on ce qu'on souhaite? On change de sentiment.

Lorsqu'on entrevoit de loin quelque bien, il fixe d'abord les désirs; lors'qu'on l'atteint, on en sent le néant. Notre âme, dont il arrêtoit la vue dans l'eloignement, ne sauroit plus s'y reposer, quand elle voit

au-dela: ainsi l'amitié, qui de loin bornoit toutes nos prétentions, cesse de les borner de près; elle ne remplit pas le vide qu'elle avoit promis de remplir. Elle nous laisse des besoins qui nous distraient et nous portent vers d'autres biens: alors on se néglige; on devient difficile; on exige bientôt, comme un tribut, les complaisances qu'on avoit d'abord recues comme un don. C'est le caractère des hommes de s'approprier peu-á-peu jusqu'aux grâces qu'on leur fait: une longue possession accoutume naturellement à regarder comme siennes les choses qu'on tient d'autrui; l'habitude persuade qu'on a un droit naturel sur la volonté des amis: on voudroit s'en former un titre pour les gouverner. Lorsque ces prétentions sont réciproques, comme il arrive souvent, l'amour-propre s'irrite, crie des deux côtés, et produit de l'aigreur, des froideurs, des explications amères, et la rupture.

On se trouve aussi quelquefois des défauts qu'on s'étoit cachés; ou l'on tombe dans des passions qui dégoûtent de l'amitié, comme les maladies violentes dégoûtent des plus doux plaisirs: aussi les hommes extrêmes, capables de donner les plus fortes preuves de dévouement, ne sont pas les plus capables d'une constante amitié. On ne les trouve nulle part si vives et si solides, que dans les esprits timides et sérieux, dont l'âme modérée connoît la vertu. Le sentiment doux et paisible de l'amitié soulage leur cœur, détend leur esprit, l'élargit, les rend plus confians et plus vifs; se mêle à leurs amusemens, à leurs affaires, et à leurs plaisirs mystérieux: c'est l'âme de toute leur vie.

ON POLITENESS.

A Translation from the French of Madame Lambert, M. Duclos, and L' Abbe Trublet.

Politeness is a Desire to please the Persons with whom we are obliged to live, and to act so that every

Body may be pleased with us; our Superiors, with our Respects; our Equals, with our esteem; and our Inferiors, with our Kindness. In short, it consists in a careful Attention to the art of pleasing, and of speaking to every one that which is agreeable to him. It attaches value to good Qualities, it makes them sensible that it acknowledges their Superiority. When you know how to exalt them, they will in their Turn cause you to be valued. They will bestow on you, above others, the Place that you wish to resign to them: This is the Interest of their Self-Love. The greatest Part of the world require pleasing Manners; but when you have none of them, you must have double the quantity of your other good Qualities. You must possess much Merit to break through clownish Manners. You must also not suffer too much Attention to yourselves to appear. A polite Person never finds any Time to speak of himself.

Politeness is a Desire to please: Nature imparts it, Education and the World increase it. Politeness is a Supplement of Virtue. In times of the greatest rudeness, in which virtue had the greatest sway, we knew less Politeness. She has come with Voluptuousness: She is the Daughter of Luxury and Delicacy. People have thought it doubtful whether it contains more Vice than Virtue. Without daring to decide on this or determine it, I believe it is one of the greatest Bonds of Society, since it contributes the most to Peace: It is a Preparative to Charity, an Imitation of Humility itself. True Politeness is modest; and as it seeks to please, it knows that the Means to succeed in that attempt are to make it appear that we do not prefer ourselves before others; and that we give them the first Rank in our Esteem. It is the Art of agreeably reconciling what is due to others, with what we owe to ourselves: For these duties have their Limits, to transgress which is Flattery to others, and Pride to ourselves. It is a Quality the most seducing.

Politeness is the Expression or Imitation of social Virtues. It is the Expression of them, if it is true; and it is the Imitation of them, if it is false. The social Virtues are those which render us useful and agreeable to those with whom we have to live. A Man who would possess all these, would necessarily have Politeness in the highest Degree. But how happens it, that a Man of lofty Genius, a generous Heart, and exact Justice, fails in Politeness, whilst we find it in a man, who is shallow, covetous, and of doubtful integrity? It is because the former is deficient in some of the social Qualities, such as Prudence, Discretion, Reserve, and Indulgence to the Defects and Weaknesses of Men. One of the first social Virtues, is to endure in others what we would prohibit to ourselves. When, on the contrary, the Second, without possessing any Virtue, has the Art of imitating them all. He knows how to testify Respect to his Superiors, Kindness to his Inferiors, Esteem to his Equals; and to persuade them that he thinks advantageously of them, without having any of the Sentiments which he imitates. We do not even at the present Time require them; and the Art of counterfeiting them is that which constitutes the Politeness of our Days. This Art is often ridiculous and low enough to be given for what it is, that is to say, for Falsehood.

Men owe Respect to each other, since Gratitude is due to all. They owe reciprocally a Politeness worthy of themselves, made for some thinking Beings, and varied by the different Sentiments which ought to inspire it. Thus the Politeness of the Great should be Humanity; that of Inferiors, Gratitude, if the Great merit it; that of Equals, Esteem and mutual Services. The most unhappy Effect of the Use of Politeness, is that of teaching the Art of doing without the Virtues that it imitates. When we are inspired in our Education with Humanity and Beneficence, we shall have Politeness, or we shall have no more Need of it. If we

have not that which announces itself by its Graces we shall have that which announces the honest Man and the Citizen,—we shall not need to have Recourse to Falsehood. Instead of being cunning to please, it will suffice to be good; instead of being false to flatter the Weakness of others, it will suffice to be indulgent. Those with whom we shall have such intercourse, will neither be puffed up nor corrupted; they will only be grateful and will become better.

Politeness consists also in delivering ourselves with a good Grace in Conversation, in not having a greater portion of wit than those into whose company we may come, in not affecting too much exactness, in occasionally giving place to Contradiction and Criticism, in a Word, in not being always rational. It is not sufficient not to say or do any Thing that may injure others; we must, in addition to this, suffer what they may say or do that is offensive or unpolite. Thus a great Part of Politeness consists in enduring the want of politeness, and in opposing only an Example of the Contrary. To manifest to others that they offend us, is almost always to offend them.

Boldness and excessive Timidity are equally contrary to true Politeness, which requires that we speak and act with a modest and easy Air. When we can please only by employing Disguise, we must then renounce the idea of pleasing. When Truth and Politeness are in competition and we cannot reconcile them between themselves, we must sacrifice Politeness to Truth.

It is very difficult to be polite with those who do not please us; and it is the Misfortune of persons of wit, that very few People please them. The greater portion of wit we possess, the more we perceive the Defects of others. Besides, People of wit find it painful to submit to an Infinity of little Formalities, which nevertheless constitute a Part of Politeness. It is therefore more difficult to be polite when we have much wit, than when we have only a moderate share of it. But when

a Man of Spirit is polite, he is more so than any other, and in a Manner much more agreeable. Every thing is to him an Occasion of saying polite Things; and the ingenious Turn which he gives them, renders them still more flattering, provided however that we do not suspect his Sincerity.

At Mr. Fletcher's School, Armley. C. E. COLLETT.

TIMÆUS, SEU DE UNIVERSO.

MULTA sunt a vobis, et in Academicis conscripta contra physicos, et sæpe cum P. Nigidio, Carneadeo more et modo disputata. Fuit enim vir ille cum cæteris artibus, quæ quidem dignæ libro essent, ornatus omnibus: tum acer investigator, et diligens earum rerum, quæ a natura involutæ videntur. Denique sic judico, post illos nobileis Pythagoreos, quorum disciplina extincta est quodammodo, cum aliquot sæcula in Italia, Siciliaque viguisse, hunc exstitisse, qui illam renovaret. Qui cum me in Ciliciam proficiscentem, Ephesi exspectavisset, Romam ex legatione ipse decedens: venissetque eodem Mitylenis mei salutandi et visendi causa, Cratippus, Peripateticorum omnium, quos quidem ego audiverim, meo judicio facile princeps: perlibenter et Nigidium vidi, et Cratippum cognovi. Ac primum quidem tempus salutationibus, reliquum percunctatione consumsimus.—*Desunt multa.*

Quid est, quod semper sit, neque ullum habet ortum? et quod gignatur, nec unquam sit. Quorum alterum intelligentia, et ratione comprehenditur, quod unum semper atque idem est: alterum quod affert opinionem per sensus, rationis expers; quod totum opinabile est, id gignitur, et interit, nec unquam esse vere potest. Omne autem quod gignitur, ex aliqua causa gigni necesse est: nullius enim rei, causa remota, reperiri origo potest. Quocirca si is, qui aliquod munus efficere moli-

tur, eam speciem, quæ semper est eadem, intuebitur, atque eam sibi proponet exemplar præclarum opus efficiat necesse est; sui autem illam, quæ gignitur nunquam illam, quam expetet, pulchritudinem consequetur. Omne igitur cælum, sive mundus, sive quovis alio vocabulo gaudet, hoc nobis nuncupatus sit. De quo id primum consideremus, quod principio est in omni quæstione considerandum, semperne fuerit, nullo generatus ortu: an ortus sit, an ab aliquo temporis principatu ortus est, quandoquidem cernitur, et tangitur, et est undique corporatus. Omnia autem talia sensum movent: sensusque moventia quæ sunt, eadem in opinione confidunt, quæ ortum habere, gigniqûe diximus, nihil autem gigni posse sine causis. Atque "illum quidem quasi petentem huius universitatis invenire, difficile: et cum jam inveneris, indicare in vulgus nefas." Rursus igitur videndum ille fabricator tanti operis utrum sit imitatus exemplar; idne, quod semper unum et idem, et sui simile; an id, quod generatum ortumque dicimus. Atque si pulcher est hic mundus, si probus ejus artifex, profecto speciem æternitatis imitari maluit: sin secus (quod ne dictu quidem fas est) generatum exemplum est pro æterno secutus. Non igitur dubium, quin æternitatem maluerit exsequi, quandoquidem neque mundo quidquam pulchrius, neque ejus ædificatore præstantius. Si ergo generatus, ad id est, effectus, quod ratione sapientiaque comprehenditur atque innumerabili æternitate continetur. Ex quo efficitur, ut sit necesse hunc, quem cernimus, mundum, simulachrum æternum esse alicujus æterni. Difficillimum autem est in omni conquisitione rationis, exordium.

A Translation from the Latin of Cicero, being the conclusion of that curious fragment entitled,

THE DREAM OF SCIPIO.

'THEN that one, red and terrible to the earth, which 'ye call Mars. After that, the Sun is fixed almost beneath

" the middle region, the leader, chief and moderator of ' the other lights, the soul and regulator of the world, ' and of so great a magnitude that it illuminates and ' fills all things with its light. One course of Venus, ' another of Mercury, follow him, as companions. The ' Moon revolves in the lowest orb, enlightened by the ' rays of the sun. But beneath there is nothing but ' what is mortal and perishable except the souls granted ' to the race of man by the bounty of the Gods. Above ' the moon, all things are eternal. For the earth, ' which is in the middle, and the ninth, moves not, and ' is the lowest, and all heavy bodies are carried towards ' it by their tendency downward.'

" After I had beheld these things in amazement, and had recovered myself, *What is this sound*, said I, *so loud and sweet, which fills my ears?* ' It is that,' answered he, ' which joined at unequal intervals, but ' which are divided according to a fixed proportion, is ' produced by the impulse and motion of the orbs; ' which, tempering shrill with bass, makes equally varied concerts. Nor can such great motions as these be ' made in silence; and nature causes, that the uttermost parts on one side sound bass, but on the other, ' shrill. For which reason, that very high course of the ' starry heaven, the revolution of which is with greater ' swiftness, moves with a shrill and loud sound; but ' the moon, being the lowest, with a bass sound. For ' the earth, which is the ninth, remaining immoveable, ' is unalterably fixed in the lowest place, containing the ' middle part of the universe. But those eight courses ' (in two of which, Mercury and Venus, the power of ' the tone is the same,) make seven sounds, divided by spaces; which number is, in a manner, the knot of all ' things. Skilful men, having imitated these with ' strings and songs, have opened to themselves a return ' to this place; as others also have, who, being of excellent understandings, have in the course of their lives ' cultivated divine studies. The ears of men, being filled

'with this sound, have grown deaf. Nor is there any 'sense in you more dull than this: As where the Nile, 'at those places which are called Catadupa, precipi- 'tates itself from the highest mountains, the nation 'which inhabits that place is destitute of the sense of 'hearing, on account of the greatness of the noise. 'But here the sound occasioned by the quick revolu- 'tion of the whole universe is so loud, that the ears of 'men cannot be sensible of it: as ye cannot look upon 'the sun when opposite to you, your sight and sense 'being overcome by its rays.' Admiring these things, I yet, at times, cast my eyes on the earth.

"Then Africanus said, 'I perceive that thou art now 'contemplating the habitation and dwelling-place of 'men. If it seems as small to thee, as it is in reality, 'look always upon these heavenly things, despise human 'things. For what celebrity, and what glory to be 'wished for, canst thou obtain amongst men? Thou 'seest that they dwell on the earth in thinly-scattered 'and narrow places; that between these places where 'they dwell, which are like specks, there lie vast 'deserts; and that those who inhabit the earth, not 'only are so divided from each other that no constant 'intercourse can subsist between them, but part stand 'oblique to you, part behind, and part before you, from 'whom you cannot expect any glory. And thou per- 'ceivest the same earth, encircled and surrounded by 'certain things resembling girdles. Two of which, 'thou seest, are placed at the greatest distance from 'each other, and being supported at both extremities 'by the poles of heaven, are stiff with frost, and that 'middle and largest one is scorched by the heat of the 'sun. Two are habitable; the southern, in which they 'who stand, press their footsteps opposite to you, hav- 'ing nothing in common with your species;—but see, 'how small a part of this other, which ye inhabit, and 'which lies near the north, falls to your lot! For all 'the earth which is inhabited by you, being narrow at

'the poles, but broader at the sides, is a small island, 'surrounded by that sea, which on earth you call *the* 'Atlantic, *the Great Sea*, and *the Ocean*. Thou seest 'how small this is, though of so great a name. From 'these known and cultivated regions, could either thy 'name, or that of any of us, pass over this Caucasus 'which thou seest, or swim across that Ganges? Who 'in the other parts of the east, or in the furthest parts 'of the west, in the north or in the south, will hear thy 'name? Which places being cut off, thou perceivest 'in what confined and narrow places your glory will 'have to extend itself. And how long will they, who 'now speak concerning you, have the faculty of speech?

'Moreover, if the progeny of future ages should desire 'to hand down to their posterity the praises of any one 'of us, which they had received from their fathers; yet 'on account of the floods and fires of the earth, which 'it is necessary should happen at a certain time, we not 'only cannot obtain eternal glory, but not even that 'which will endure for a short time. And what does 'it concern thee, that thou shouldst be spoken of by 'those who will be born after thee, no mention having 'been made of thee by thy predecessors who were not 'fewer, and certainly better men? Especially, since 'with those very persons by whom our name may be 'now heard, none can obtain the remembrance of one 'year. For men commonly measure the year only 'by the return of the sun, that is, by the return of one 'star: but when all the stars shall have returned to 'the same place whence they set out, and shall by 'long intervals have brought back the same appear- 'ance of the whole heaven, then it may be truly called 'a returning year; in which I dare scarcely say, how 'many ages of men are contained. For, as formerly 'the sun seemed to men to be eclipsed and extinguished 'when the soul of Romulus entered these temples, so 'whensoever the sun shall be eclipsed again in the same 'part, and at the same time, then all the signs and stars

'being recalled to the same beginning, you may reckon
'it a finished year. Know that the twentieth part of
'this year has not yet passed away. Wherefore, if thou
'hast despaired of returning to this place, in which all
'things are for great and excellent men, of what value
'is that human glory which can scarcely endure through
'a very small portion of one year? If thou wishest,
'therefore, to look on high, and to behold this habita-
'tion and eternal house, neither give thyself up to the
'opinion of the common people, nor place the hope
'of prosperity in human rewards. Virtue itself must
'draw thee by its own attractions to true glory. What
'others speak concerning thee, let them take heed;
'but yet they will speak. But all their discourse is
'confined by the narrow bounds of those places which
'thou seest, nor was it ever lasting concerning any one.
'It perishes at the death of men, and is extinguished
'in the forgetfulness of posterity.'

"After he had said these things, I replied, *If indeed, Africanus, there be a road as it were to the entrance of heaven for those who have deserved well of their country, although from my childhood I have followed in the footsteps of my father and you, and have not been unmindful of your glory, yet now, so great a reward being set before me, I will strive much more vigilantly.* And he said,
'Strive indeed; and take it for granted that thou art
'not thyself mortal, but that this body is. For thou art
'not what that form declares, but the soul of every one
'is the man himself; not that form which can be point-
'ed out by the finger. Know therefore that thou art a
'God: if indeed that is God, which lives, perceives, re-
'members, and provides, who so rules, regulates, and
'moves that body, over which it is set, as the chief
'God does this world: And as the eternal God governs
'this universe from some part; so the eternal soul
'guides that frail body. For that which is always in
'motion, is eternal; but that which brings motion to
'something and which itself is moved from without,

'when it has an end of motion it has of necessity an 'end of living. Therefore that alone which moves itself, 'because it is never deserted by itself, never ceases at 'all to move. Moreover, this is the fountain, this the 'beginning of motion to other things which are moved. 'But there is no origin to the beginning. For all things 'proceed from the beginning; but itself from nothing. 'For that is not a beginning which is produced from 'something else; and if it has no beginning, it can 'never have an end. For the beginning being extinct, 'neither will it be produced from another thing, nor 'will it produce any other thing from itself; since it is 'necessary that all things should rise from the begin- 'ning. Thus it happens, that the beginning of motion 'is from that which moves of itself: But that can nei- 'ther be born nor die; it must be that all heaven and 'nature itself decay, and be at a stand, nor obtain any of 'that force which is put in motion by the first impulse.

'As it is manifest, therefore, that what moves itself 'is eternal, who will say that this nature is not to be as- 'signed to souls? For every thing is inanimate, which 'is moved by an outward impulse: but that is a living 'soul that moves by an inward motion of its own. 'For this nature and power peculiarly belong to the 'soul. If this is the only thing that moves itself, it 'cannot have been born, but is eternal. Exercise this 'in the highest concerns. The best concerns are those 'respecting the safety of thy country: by which if the 'soul be agitated and exercised, it will arrive more 'speedily at this its habitation and dwelling: and it 'will do this the more swiftly, if now, whilst it is shut 'up in the body, it look abroad, and abstract itself as much as possible from the body, in the contempla- 'tion of those things which are without. For the souls 'of those that have given themselves up to the pleasures 'of the body, and yielded themselves as the servants of 'them, and by the impulse of passions obedient to plea- 'sure, have violated the laws of the Gods and men,—

'being freed from their bodies, are carried about this 'earth; and come not into this place, until they have 'been tossed about for many ages.'—He departed and I immediately awoke."

Brunswick Place Academy, Liverpool. J. WELLSTOOD.

We received other excellent translations from Masters D. Thornley, A. Dunn, R. Christian, J. Chaffers, H. Emmet, H. Christian, and John Kendall.

Dulces ante omnia Musæ. VIRG.

SPECIMEN
OF THE
Heroic Poetry of Java.

The following beautiful verses are extracted from that very able work, *the History of Java, by Thomas Stamford Raffles*, Esqr. F. R .S. and A. S. late Lieut. Governor of that Island. It is part of an interesting Epic Poem, called *the Holy War* or rather the *War of Woe*, and is translated into English verse by the Rev. Thomas Raffles, of Liverpool.

Satia the commander of the Kurawa forces, falls in battle, and his wife *Satia Wati* goes forth among the slain to find the body of her husband.

Wearied with fruitless search, and in despair
To find the object of her pious care,
Her murder'd lord, who on the battle plain
Lay all neglected mid the thousands slain,
She drew the dagger from its sheath of rest,
Intent to plunge it in her heaving breast.
Just then, as if in pity to her grief,
Flash'd the red light'ning to the maid's relief,
And shew'd with horrid glare the bloody way
To where her husband's mangled body lay.
Another flash, indulgent from the skies,
Points to the spot where Salia's carriage lies,
And Salia's self, whom living she ador'd,
The bleeding body of her murder'd lord.
The richest flow'rs, by heav'nly influence, shed
Their sweetest odours o'er his honour'd head,

The mutt'ring thunder mourn'd his early tomb,
And heav'n in show'rs bewail'd the hero's doom.
With eager grasp the livid corpse she press'd
In frantic wildness to her throbbing breast;
Tried every art of love that might beguile
Its sullen features to one cheerful smile;
Kiss'd those dear lips so late of coral red,
As if unconscious that the soul had fled;
Then in her folded arms his head she rais'd,
And long on those beloved features gaz'd.
With Siri-juice his pallid lips she dy'd,
And to his wounds its healing balm applied;
While with the skirt of her embroider'd vest,
She wip'd the blood-drops from his mangled breast.
" Ah! then, my princely lord, whom I have found
" Bleeding and mangled on this cursed ground!
" Why are thy lips in sullen silence seal'd
" To her who sought thee on this battle field?
" Wilt thou not speak—my love, my lord, my all,
" Or still in vain must Satia Wati call!
" Say, shall my copious tears in torrents flow
" And thus express my agony and woe?
" How shall I move thee, by what art beguile
" The ghastly air of that unmeaning smile?"
Thus soft and tender were the words she pour'd,
To move the pity of her murder'd lord;
But ah! no sound the unconscious dead return'd,
No fire of love within his bosom burn'd;
While at each pause a death-like stillness stole
O'er the deep anguish of the mourner's soul.
" And was it thus to bow thy honour'd head
" Amid the thousands of the mingled dead,
" That on that fatal morning thou didst glide
" With gentle footsteps from thy consort's side?
" And thus to reach the glorious realms above
" Without the faithful partner of thy love?
" But earth has lost its fleeting charms for me,
" And, happy spirit, I will follow thee!

" Oh! meet and bear me o'er that fatal stone,
" Nor let me pass it, trembling and alone.
" Though Widadaris shall obey thy call,
" Yet keep for me a place above them all.
" To whom but me does that first place belong,
" Who sought and found thee mid this ghastly
" throng?

CHARACTER OF OUR KING AND QUEEN.

At a time when both these illustrious personages are interesting sufferers, near the verge of life, and insensible to human applause, it cannot but gratify every friend to his sovereign to contemplate the noble endowments of the mind, and the exquisite moral taste, which are displayed in the interview of their Majesties with the justly celebrated Doctor Beattie, the successful champion of Truth in opposition to scepticism and error; of that interview the Doctor himself gives the greater part of the following pleasing account:—

If any thing could add to the satisfaction he naturally felt from this fortunate conclusion of his affairs, it was the distinguished honour he met with, before he left London, of a personal and private interview with his Majesty, at the palace at Kew.

Dr Beattie had been informed by Dr Majendie, who lived at Kew, and was often at the palace, that the King having asked some questions of the Doctor respecting him, and being told that he sometimes visited Dr Majendie there, his Majesty had desired to be informed the next time Dr Beattie was to be at Kew. What his Majesty's intentions were, Dr Majendie said he did not know; but supposed the King intended to admit him to a private audience. A day was therefore fixed, on which Dr Beattie was to be at Dr Majendie's house, early in the morning, of which the Doctor was to give notice to his Majesty. Of this interesting event, so honourable to Dr Beattie, I shall transcribe, in his own words, the account he has given in his Diary.

"Tuesday, 24th August, set out for Dr Majendie's at Kew-Green. The Doctor told me, that he had not seen the King yesterday, but had left a note in writing to intimate, that I was to be at his house to-day; and that one of the King's pages had come to him this morning, to say, "that his Majesty would see me a little after twelve." At twelve, the Doctor and I went to the King's house at Kew. We had been only a few minutes in the hall, when the King and Queen came in from an airing, and as they passed through the hall, the King called to me by name, and asked how long it was since I came from town. I answered, about an hour. "I shall see you," says he," in a little." The Doctor and I waited a considerable time, (for the King was busy,) and then we were called into a large room, furnished as a library, where the King was walking about, and the Queen sitting in a chair. We were received in the most gracious manner possible, by both their Majesties. I had the honour of a conversation with them, (nobody else being present, but Dr Majendie,) for upwards of an hour, on a great variety of topics, in which both the King and Queen joined, with a degree of chearfulness, affability, and ease, that was to me surprising, and soon dissipated the embarrassment which I felt, at the beginning of the conference. They both complimented me, in the highest terms, on my "Essay," which, they said, was a book they always kept by them; and the King said he had one copy of it at Kew, and another in town, and immediately went and took it down from a shelf. I found it was the second edition. "I never stole a book but one," said his Majesty, "and that was yours; (speaking to me) I stole it from the Queen, to give to Lord "Hertford to read." He had heard that the sale of "Hume's Essays" had failed, since my book was published; and I told him what Mr Strahan had told me in regard to that matter. He had even heard of my being in Edinburgh, last summer, and how Mr Hume

was offended on the score of my book. He asked many questions about the second part of the "Essay," and when it would be ready for the press. I gave him, in a short speech, an account of the plan of it; and said, my health was so precarious, I could not tell when it might be ready, as I had many books to consult before I could finish it; but, that if my health were good, I thought I might bring it to a conclusion in two or three years. He asked, how long I had been in composing my "Essay?" praised the caution with which it was written; and said, he did not wonder that it had employed me five or six years. He asked about my poems. I said, there was only one poem of my own, on which I set any value, (meaning the "Minstrel,") and that it was first published about the same time with the "Essay." My other poems, I said, were incorrect, being but juvenile pieces, and of little consequence, even in my own opinion. We had much conversation on moral subjects; from which both their Majesties let it appear, that they were warm friends to christianity: and so little inclined to infidelity, that they could hardly believe that any thinking man could really be an atheist, unless he could bring himself to believe, that he made himself; a thought which pleased the King exceedingly; and he repeated it several times to the Queen. He asked, whether any thing had been written against me. I spoke of the late pamphlet, of which I gave an account, telling him, that I never had met with any man who had read it, except one Quaker. This brought on some discourse about the Quakers, whose moderation, and mild behaviour, the King and Queen commended. I was asked many questions about the Scots universities, the revenues of the Scots clergy, their mode of praying and preaching, the medical college of Edinburgh, Dr Gregory, (of whom I gave a particular character,) and Dr Cullen, the length of our vacation at Aberdeen, and the closeness of our attendance during the winter, the number of students

that attend my lectures, my mode of lecturing, whether from notes, or completely written lectures; about Mr Hume, and Dr Robertson, and Lord Kinnoull, and the Arch-bishop of York, &c. &c. &c. His Majesty asked what I thought of my new acquaintance, Lord Dartmouth? I said, there was something in his air and manner, which I thought not only agreeable, but enchanting, and that he seemed to me to be one of the best of men; a sentiment in which both their Majesties heartily joined. "They say that Lord Dartmouth "is an enthusiast," said the King, "but surely he says "nothing on the subject of religion, but what every "christian may, and ought to say." He asked, whether I did not think the English language on the decline at present? I answered in the affirmative; and the King agreed, and named the "Spectator" as one of the best standards of the language. When I told him that the Scots clergy sometimes prayed a quarter, or even half-an-hour, at a time, he asked, whether that did not lead them into repetitions? I said, it often did. "That," said he, "I don't like in prayers; and excel- "lent as our liturgy is, I think it somewhat faulty "in that respect." "Your Majesty knows," said I, "that three services are joined in one, in the ordinary "church-service, which is one cause of those repeti- "tions." "True," he replied, "and that circumstance "also makes the service too long." From this, he took occasion to speak of the composition of the church-liturgy; on which he very justly bestowed the highest commendation. "Observe," his Majesty said, "how flat those occasional prayers are, that are "now composed, in comparison with the old ones." When I mentioned the smallness of the church-livings in Scotland, he said, "he wondered how men of liberal education would chuse to become clergymen there," and asked, "whether in the remote parts of the country, the clergy, in general, were not very ignorant?" I answered, "No, for that education was

"very cheap in Scotland, and that the clergy, in "general, were men of good sense, and of competent "learning." He asked, whether we had any good preachers at Aberdeen? I said, yes, and named Campbell and Gerard, with whose names, however, I did not find that he was acquainted. Dr Majendie mentioned Dr Oswald's "Appeal," with commendation; I praised it too; and the Queen took down the name, with a view to send for it. I was asked, whether I knew Dr Oswald? I answered, I did not; and said, that my book was published before I read his; that Dr O. was well known to Lord Kinnoull, who had often proposed to make us acquainted. We discussed a great many other topics; for the conversation, as before observed, lasted for upwards of an hour, without any intermission. The Queen bore a large share in it. Both the King and her Majesty showed a great deal of good sense, acuteness, and knowledge, as well as of good nature and affability. At last, the King took out his watch, (for it was now almost three o'clock, his hour of dinner,) which Dr Majendie and I took as a signal to withdraw. We accordingly bowed to their Majesties, and I addressed the King in these words: "I hope, "Sir, your Majesty will pardon me, if I take this "opportunity to return you my humble and most "grateful acknowledgments, for the honour you have "been pleased to confer upon me." He immediately answered, "I think I could do no less for a man, "who has done so much service to the cause of Chris-"tianity. I shall always be glad of an opportunity "to shew the good opinion I have of you." The Queen sat all the while, and the King stood, sometimes walking about a little. Her Majesty speaks the English language with surprising elegance, and little or nothing of a foreign accent. There is something wonderfully captivating in her manner, so that if she were only of the rank of a private gentlewoman, one could not help taking notice of her, as one of the most

agreeable women in the world. Her face is much more pleasing than any of her pictures; and in the expression of her eyes, and in her smile, there is something peculiarly engaging. When the Doctor and I came out, "Pray," said I, "how did I behave? "Tell me honestly, for I am not accustomed to con- "versations of this kind." "Why, perfectly well," answered he, "and just as you ought to do."—"Are "you sure of that?" said I.—"As sure," he replied, "as of my own existence: and you may be assured of "it too, when I tell you, that if there had been any thing "in your manner or conversation, which was not per- "fectly agreeable, your conference would have been "at an end, in eight or ten minutes at most." The Doctor afterwards told me, that it was a most uncommon thing for a private man, and a commoner, to be honoured with so long an audience.

The Difference of Style in ADDISON and JOHNSON.

(continued from page 180.)

JOHNSON's copiousness often degenerates into the florid: Addison's simplicity very rarely assumes the appearance of weakness.—In Johnson may be perceived almost a perpetual tendency to verbosity; and from this cause it is that he seems occasionally to be amused by noise without meaning. Addison can amplify when he finds it necessary, but conciseness was his great excellence; and, in studying this, in some exceedingly few instances he has not escaped the charge of obscurity.—Had both of them been requested to write on some one trite subject, Addison would have dismissed it speedily, exemplifying a certain ease and gracefulness in its expression; while, in the hands of Johnson, it would have been involved in so many folds, and presented in such different forms, as scarcely to be recognised as consisting of any thing like the same sentiments.

Johnson's flow of language is like the majestic rolling of a large and undulating stream: Addison's carries with it the resemblance of a river deep and clear.—Addison surpassed in terseness of phraseology: Yet Johnson shewed himself a complete master of language, by being both pithy and concise when the subject in which he was engaged required the display of those qualities.—On becoming acquainted with Johnson, and after your ear is accustomed to his cadence, you sometimes find it tedious, especially when he is negligent in the proper employment of variety in the construction of his sentences. In this respect, again, the superiority is usually adjudged to Addison.

Till Addison's writings appeared, it scarcely entered into the apprehension of any one that the English tongue was capable of so much purity, sweetness, and genuine pathos: And before Johnson wrote, it was thought destitute of native dignity and grandeur.—Indeed, it would puzzle a good philosopher to determine which of the two has been the greatest literary benefactor to his country. The occupations assigned to them in the philosophy of language have been very dissimilar: It was Addison's department to cultivate and refine; Johnson succeeded to him, and his province may be said to have been the enrichment of speech.

Addison's style is well adapted to philosophical disquisitions, logical reasoning, history, narrative and epistolary compositions. Johnson's style is properly directed when it is applied to the hortatory part of sacred eloquence, to pathetic description, popular oratory, and tragic composition. Addison's eloquence is natural, and contains nothing forced. It always recommends itself to persons of taste and genius, because its entire beauties can be appreciated by them alone. Were an angel deputed to convey much interesting information to a few favoured individuals of masculine understandings and capacious minds, it is probable that he would adopt the language of Addison. But if on a

momentous subject he were commissioned to address a multitude of various acquirements and capacities, and if pity exerted any of its genial sway over his determinations, it would influence him to employ the style of Johnson on account of the amplification which it allows, and the agreeable reiteration of the same sentiments.

That Johnson's style is unfit for some purposes, may be demonstrated by his epistolary correspondence. In his letters to Mrs. Thrale and others, his attempts at ease and pleasantry are unbending efforts, and not unlike the clumsy gambols of an unwieldy elephant. They are stiff and formal compositions, and not in the least worthy of so great a man. As a most charming exception, however, must be noticed the manner in which his condolence or sympathy is expressed. His letters of consolation are admirable specimens of his powers, and distinctly point out the character of writing to which his style is adequate.—But how appositely employed does Addison's style appear to be in his letters! They are the simple, unconstrained, and familiar effusions of a polished mind; and may serve as finished models for the imitation of those who desire to excel.

From these observations, it is to be understood, that the excellences attributed to each of our great authors, are not exclusively his own, but are possessed, though in an inferior degree, by the other. It would not be a work of difficulty to produce a few of the acknowledged productions of both of them, in which few men would venture to pronounce which is the work of Addison, and which that of Johnson.

In this enumeration of their respective beauties, (for their defects are trifling,) commendation seems to preponderate in favour of Addison. But let it be remembered that though Johnson's errors are the more palpable and numerous, the tribute of applause due to him is greater. What Milton achieved for our poetry, Johnson performed in behalf of our prose,—he assisted to adorn and elevate it. He that blames Johnson for

the introduction of new and uncommon words, shews himself but imperfectly acquainted with the usages of our early prose writers, especially of those who flourished from the reign of Edward the Seventh till the time of our Second James. Johnson has altered the terminations of many words that were the current literary speech of those days; but the alteration was evidently made for the sake of cadence,—a point in his compositions which he particularly regarded, and in the correct observance of which almost all his imitators have failed. To a modern and uninstructed ear such words might have a novel sound; but the greater portion of them before Johnson's days constituted a part of our language. A few, indeed, were the creatures of his own invention; but I can see no reason why he ought not to be indulged in the same latitude as others among his predecessors and his co-temporaries. The formation of a style peculiar to himself was the effort of a great mind. When it became matured and chastened, it was as admirable a trophy as could grace the arms of any conqueror. Had his expressions been employed by an inferior master in the glorious art of composition, destitute of his direction they would have wanted masculine firmness and would have evaporated into declamatory bombast and towering insignificance. —A fault commonly attributed to him is that of his dwelling long on a single idea, and giving it nearly every possible turn of variation. Yet to impart animation and interest to such apparent tautology, requires the exertion of great talents: And uncommonly great were Johnson's talents, even when he exercised them in mere trifles.———An allowable cause for much of his negligence, is to be found in the haste in which his poverty obliged him to compose. He did not attain to any thing like competency till late in life: And penury, it is well known, is not a soil the most genial for the expansion of thought or the luxuriance of intellect. Johnson's own remark, of the necessity

of indigence in his case to draw the least valuable labour from him, does not invalidate my reasoning, but may afterwards be made available to another purpose. The advantages of Addison were in this particular much superior: For his days were passed in comparative affluence. His natural timidity also inspired him at many times with an unnecessary dread of saying or writing any thing that would be derogatory to his reputation. The delicacy of his taste had the same tendency. Johnson, on the contrary, in all the overbearing confidence of his natural disposition, when forced by want, committed to the press the uncorrected labour of a few moments, and relied for its success more on a knowledge of the competency of his own powers, than on a consciousness of any great share of merit in his performance. These casual rapid sketches, or unpolished rough-draughts, operated as foils, and shewed off the chaste productions of his studious hours to more advantage.

From these two great, distinct, and admirable modes of writing has arisen the beautiful style of some of our most elegant modern authors. It is a felicitous combination, and well becoming this the Augustan æra of British literature. I stated in the former part of this paper, (page 178,) that literary men are influenced in the variety of their styles by their partiality for some favourite authors. I confess myself to have a predilection in this case; but it is not in favour either of Addison or Johnson. My favourite is Swift. Never did any man's works appear before the world with greater disadvantage, than did those of Swift. The plight in which they have generally been exhibited is most pitiable. Even his most admired pieces were anonymous, and had to creep into notice by means of their own intrinsic worth, independent of the celebrity of their author. Mutilated and imperfect copies were surreptitiously obtained, and published in distant parts with all the imperfections which they had received in

the transfer and the addition of others. A very small portion indeed received the last correction of the author: In the voluminous collection of his writings, it is therefore nothing unusual to find a note appended to the close of every fourth or fifth page, in which the editor with the greatest propriety informs his readers that some particular sentence in that page is not grammatical. The most egregious of these errors may principally be attributed to the haste with which he generally wrote, and his distance from the press. Notwithstanding all these untoward circumstances, Swift will always by impartial judges be accounted an able and nervous writer. Without any adventitious aid, he has shewn several of the finest capabilities of our native English in the structure of his sentences and the management of his style. I am no admirer of the extreme pruriency of his wit. I would neither venture to defend his acrimony in politics, nor offer any apology for his intolerance in religion. But I must nevertheless be permitted to consider him no mean contributor to the treasury of the English language. JUDEX.

A BRIEF MEMOIR

OF THE

Life of the late Mr. John Ryley,

TEACHER OF THE MATHEMATICS IN LEEDS.

(Concluded.)

It would be a very agreeable amusement to me, and may become the subject of a future paper in the Leeds Correspondent, to trace out the expanding acquaintance of his mind, and the increased grasp of his powers, in respect to mathematical subjects. And this may easily be ascertained, by observing the advance in his communications to the Diaries, and other periodical works of the same kind. On a very cursory inspection

it is evident, that Mr. Ryley was no mushroom adept,

That over-night in haste arose,
And lost his moisture in an hour.

His application must have been intense and continued; for in a comparatively short period, few mathematical difficulties presented themselves with which he was not able to cope in one shape or other.

His residence at Beeston may be regarded as a period of his life in which he had a great portion of enjoyment. His intimacy with the very learned Curate of Beeston, the Rev. Joseph Whiteley, contributed not a little to his comfort and improvement. This gentleman, who had taken his first degree with high honour at Cambridge, was greatly his superior in mathematical attainments. I never knew a person of true intellectual eminence that was more ready to communicate instruction, or that had a happier method of doing it, than Mr. Whiteley. Blessed with great kindness of disposition and urbanity of manners, he was always in his element when in the act of imparting knowledge or conferring a benefit. Possessing an exquisite perception of what was grand and beautiful in composition, he instructed his pupils in the principles of good taste; and his own style, to those who were observant of it, presented an admirable pattern of excellence. Living in habits of friendship with this benevolent and enlightened Clergyman, Mr. Ryley would have been negligent indeed had he not profited by his advice and example.

In the year 1789, I find this reverend gentleman uniting with a number of the most respectable of his parishioners, in a recommendation of Mr. Ryley, as a proper person to succeed to the situation of head-master of the Blue-coat school in Leeds, which was then vacant. The following is a copy of that document;

"BEESTON, June 24th, 1789.

'We, whose names are here under-written, do tes-
'tify that John Ryley hath served the office of School-
'master in Beeston for the space of thirteen years last

'past: That, within that term, he has, so far as we 'know, given general satisfaction: That his wife has 'been accustomed, most of that time, to teach knitting 'and plain sewing: And that the said John Ryley is 'possessed of abilities far above his present situation, 'and very superior, we conceive, to those which are 'necessary for a Master of the Charity-School in 'Leeds.

'JOSEPH WHITELEY, Curate of Beeston.
'JOHN HARE, Chapel-warden.
'NATHANAEL JOHNSON.
'J. WILKES.'

The Rev. Mr. Whiteley's interest with the Trustees of that Charity, and the knowledge which many of them had of Mr. Ryley's character and acquirements, rendered all opposition to his election ineffectual.

In his removal to Beeston, he had reached the summit of his desires. But now, to be acknowledged the governor of seventy charity-boys, was to him an additional grand acquisition: And the yearly fixed stipend of Seventy Pounds was like the opening of the king of Lydia's treasures.—Nor was the charity-school the only source of his income. For the hours of his attendance in the school being but six, the remaining portion of each day was well filled up with private tuition at the residence of different gentlemen in the town, or by sets of pupils who attended twice or thrice in each week at his own house, either early in the morning, at the hour of noon, or in the evening.—The Blue-coat school in Leeds was at that period conducted on the plan of many other public schools, in which, according to the well-intended statutes of their founders, all the Saint-days in the Calendar, or, as school-boys designate them, *the red-letter days*, were observed as holydays. Though the number of these days of relaxation was considerable, Mr. Ryley had scarcely any portion of them unoccupied, so much were his instructions in request.

Many of the youths with whom he had engagements on these occasions, were intended for the University of Cambridge, at which celebrated seat of learning, nearly all public honours are adjudged to mathematical excellence and proficiency. It becomes necessary, therefore, that all who are laudably emulous of academic distinction should be intimately acquainted with the rudiments of the mathematics.

Mr. Ryley bestowed uncommon pains on such of them as applied to him for information, and contributed all that was in his power towards their improvement. Concerning many of those who had made a creditable progress while with him, he entertained a fond anxiety; and felt a commendable emotion of complacency, when, after a public examination in the Senate-house, he had the satisfaction to learn that the impartial and rigid trial of their mental prowess had issued favourably, and that they had been crowned with the choicest honours which the University could bestow.

To the credit of several of these young men let it be recorded, that they were extremely sensible of the obligations which they owed to his judicious advice and direction in the commencement of their mathematical career; which obligations, they were aware, could never be cancelled by the mere sordid payment of a paltry sum of money for attendance given and duties performed. Between him and the most judicious of them, there subsisted something beyond the mercenary bond of *employer* and *employed*. They were alive to the more legitimate and powerful claims of gratitude and affection. His library, which was very select, exhibited not a few valuable scientific works that had been presented to him by his scholars, who, in the hour of their literary triumph, considered the study of their old tutor as the most appropriate temple in which to suspend some pleasing votive trophy, there to remain at once a monument of their own success and gratitude, and a compliment to his genius and virtues.

Soon after his settlement at Leeds, his worth, which was hidden under a cloud of bashfulness, began to be noticed by that acute observer of mankind, Benjamin Gott Esquire. Being himself one of the best mathematicians in the North of England, he was well qualified to appreciate Mr. Ryley's talents. The favour of this gentleman proved to be of the greatest consequence to his rising prospects. He not only engaged Mr. Ryley to devote some of his leisure hours to the instruction of a part of his family; but he gave him the benefit of his influence in every quarter where it could be available.

It was some time about the year 1800, that the Trustees of the Free Grammar-school in this town were induced to employ Mr. Ryley, four evenings and two afternoons in each week, as well as on all the saint-days, to instruct the youth in Arithmetic, Mensuration, Algebra, and other branches of general mathematical knowledge. With the assistance of his son John, a most accomplished young man, he was enabled to perform this arduous service with tolerable satisfaction to himself and immense benefit to the institution. I was at that period a free-school boy, and can therefore speak the more confidently respecting the utility of the course of instruction which was then pursued. The school became crowded, there soon being as many pupils as both the upper and lower forms could conveniently accommodate. The masculine branches of learning which were introduced and cultivated under the superintendance of Mr. Ryley, broke through the monotony, and relieved the tædium, of the other studies of the place. Mathematics agreeably succeeded to Classics; and these, in their turn, took precedence of the former; while each of them, no longer viewed as a tiresome and unceasing task, operated rather in the form of a pleasing relaxation.

But the two classical masters,—thinking it due to themselves and their successors to suffer no innovation

in the school, either as it concerned the appropriation of its funds or the course of instruction,—sought relief from the Lord Chancellor, who decided the question in their favour. In consequence of this adjudication, Mr. Ryley discontinued his attendance; and the attempt, to combine a mathematical with a classical education at the Free Grammar School, has not since that time been resumed. He had, however, as much private tuition, as his leisure would allow him to devote to that purpose.

Many gentlemen of the first eminence as mathematicians, in passing through Leeds, have indulged themselves in the pleasure of paying their personal respects to Mr. Ryley at his residence near the Charity-school. Those of them who have any recollection of the site of that building, will know that the first room of the first floor, into which they entered, was nearly three feet below the level of the surrounding earth, and was consequently very damp and unhealthy. The small parlour beyond the first room, was equally low; and it was in this the dampest of these damp rooms, that Mr. Ryley during a period of five-and-twenty years, spent the greater portion of his studious hours, before school-time in the morning, after five o'clock in the evening, and often till midnight. Many hours of the summer months I have passed with him in that recess, which was then cool and dangerously pleasant. It must have been excessively prejudicial, even to a hardy constitution, to remain only two hours in a room, that, during winter, had scarcely at any time its surface completely free from damp, although the moisture was attempted to be kept-down by the aid of a good Yorkshire fire.—No one will hesitate to attribute, to Mr. Ryley's sedentary occupations in such a room, the gouty and rheumatic habit of body, from which he suffered much in the closing scenes of life. The severe paroxysms under which he occasionally laboured, were much relieved by his resorting to Buxton, whenever that was practicable. When he neglected this precaution, he was

generally confined many months afterwards to his room. In the midst of his most excruciating pains, when to feel the least motion in the room was a burden to him, his darling amusement was the solution of a difficult problem, or some other hardy mental exercise suited to his powers. In the attention which such studies required, his faculties were absorbed; and this abstraction of thought caused him to feel a temporary suspension of bodily suffering, which was in his case at all times desirable. Those only who have themselves felt the distressing pains of gout, can have any moderate conception of their dreadful intensity. Mr. Ryley bore them, as they occured at intervals, for a series of years, with the firmness of a man and the patience of a philosopher. But though he courageously endured them, they gradually undermined his hale constitution, and at last prematurely involved it in the ruin of death.

Nearly three weeks prior to his dissolution, his old enemy the gout made another desperate attack. His robust frame was worn down; and he had no adequate physical force to justify the idea of an effectual resistance. From the first day of his indisposition, he was evidently conscious that a general decay of nature had ensued, and that every symptom of his disorder was unfavourable. Yet he preserved his equanimity of mind; and was particularly desirous to create no painful alarm respecting him in any branch of his family. In imitation of every good man, he began seriously to prepare himself for the worst that could befal him, and spent much of his time in private acts of devotion. Many expressions which he uttered, denoted the religious tendency of his affections: They are treasured up with mournful pleasure in the memories of his sorrowing relatives.—At last on the 22nd day of April, 1815, the Gout seized on his vitals, and on the same day he expired, in the 69th year of his age.

He was blessed with a family of seven children, three sons and four daughters. Of these, two daughters died in infancy; and his second son John, who has already

been mentioned, died on the 8th of September, 1809, in the Twenty-fifth year of his age. He possessed uncommon abilities and a fine taste. Indulging his native propensity to literature, he chose the genial profession of a bookseller; and by his industry and engaging manners was very successful in business.—Mr. Ryley's other four children survive him. The eldest son is engaged in the woollen cloth trade; the second is a thriving butcher, a young man highly deserving of encouragement. One of his daughters keeps a flourishing school at Dewsbury; the other resides at home with her mother.

Unlike many of the would-be philosophers of the day, Mr. Ryley was a believer in the christian revelation: And he has been heard to declare, that the benefit which he derived from the doctrines of the Bible was a great comfort to his spirit under all the adverse circumstances of life. In profession he was a member of the Church of England. Yet Bigotry had no part in him. He lived on terms of the greatest intimacy with many excellent men who were Dissenters. His attachment to the Rev. Thomas Langdon, the worthy minister of the Baptist Congregation in this town, was very great. To this gentleman he considered himself under incalculable obligations for the friendly attention and the spiritual aid which he had afforded to his son John, under the frequent and severe attacks of the complaint which brought him to the grave.

In stature Mr. Ryley was remarkably stout. He was upwards of six feet in height. Nature had done much for his exterior in every respect except in that of his countenance, which from his fixed habits of close thinking exhibited at first sight somewhat of a repulsive appearance. Lavater's laws of physiognomy were, however, inapplicable to him; for seldom did such an uncouth aspect cover a nobler or more benevolent mind. You had only to converse with him when all your former conceptions of him vanished; and you instantly

perceived and acknowledged in him all the genuine marks of a modest, clever, and communicative person. To those readers who never saw him, I cannot convey a better idea of the man than by saying that in his form, size, and countenance, he was not much unlike Doctor Samuel Johnson: And I know of no likeness of that great man which would be more befitting Mr. Ryley than the celebrated one in which the Doctor is represented in the posture of a Hercules leaning on a club.—Other points of resemblance might be observed between Doctor Johnson and Mr. Ryley. Nothing of this kind was more apparent than the hasty and nervous enunciation of their sentiments on any interesting topic that arose in course of conversation. Mr. Ryley made his remarks in as full and sonorous periods, and with as much manly firmness, as Johnson could have done. Some persons would on these occasions have given the preference to the Doctor, on account of his not expressing himself in broad Yorkshire, which was Mr. Ryley's favourite dialect. J. N.

ANSWERS TO THE QUERIES

Proposed in the Third Number of the Second Volume.

(1.) What part of Speech is the Word *to*?

By Mr. John Baines, Nottingham.

To analyse a Sentence with accuracy, its Meaning must be perfectly understood; otherwise it would be impossible to parse those Words which in different Situations constitute different Parts of Speech. This Observation, however, only extends to Words which are capable of becoming different Parts of Speech. For, although Grammarians and Writers of Dictionaries have marked the Word "to" both as a Preposi-

tion and an Adverb, yet I think it cannot be accounted the latter with any Degree of Propriety, except when it is taken into Composition, as in the Words, *hitherto, heretofore,* &c. and then it is not an Adverb by itself. Moreover, an Adverb must answer to one or other of the Questions, How? How much? When, or Where? of which the Word "to" is incapable by itself; and therefore I hesitate not to assert that the word "to" is a Preposition in all the given sentences.

(2) On the tails of comets.

By Mr. J. D. Sollitt.

Amongst the numerous points in speculative philosophy, that have engaged the attention of the philosophical observer, there is not one which is involved in so great uncertainty, and on which so little has been determined, as upon what relates to the nature and properties of Comets. For whilst some of our observations tend to establish the hypothesis of Newton, and appear to prove that comets are composed of dense and solid matter; other observations tend to prove the contrary opinion. Whilst their regular motions round the sun in their respective orbits, make them appear to have somewhat of the nature of the other planetary bodies, observations upon the comets themselves, in a great measure contradict this supposition. And perhaps nothing has a greater tendency to do this, than the observations which were made, by myself, in conjunction with others, on the last great comet of 1811. In the first place, we could never observe any thing like a well defined-disc, not even when assisted by our best telescopes. And in observing the tail of the comet when it passed over stars of the third and fourth magnitude, we could not perceive that they lost any considerable degree of their lustre. Which observations evidently prove, that the tail and nucleus of a comet have not that degree of density which we find in the atmospheres and bodies of the planets.

This hypothesis likewise receives further support from the observations which were made on the first great comet of 1770 which was observed to pass and repass through the very middle of the moons of Jupiter without occasioning the least disorder among them. To these remarks, I may add, that while some of the bodies of comets that have been observed, have appeared well-defined, other comets have been seen in which there could not be distinguished the least visible nucleus; and it has been further observed, that the bodies of comets are of all variable degrees of density, from matter the density of which is equal to that of the other planets, up to that of a gas, or thin elastic fluid. Astronomers have likewise observed, that the bodies of some comets during the time of their appearance have been subject to the most surprising changes; sometimes appearing round and well-defined, and at other times appearing broken and irregular; which are evidently changes that could not take place if the comets were of the same nature as the planets.

From these observations, together with all that I have yet met with upon the nature and properties of comets, I think the supposition most reasonable that the comets are incipient worlds, passing from the gaseous state, and are deriving solidity from the precipitation and condensation of the particles of matter which compose them. By allowing this theory to be correct, we at once account for all the various appearances which are visible in those singular bodies, from those that appear as nothing but an accumulation of gaseous matter, unto those that have nearly arrived at the solid state. But succeeding observations on comets, in which it may be possible to distinguish the different stages of chaos, and progressive formation, can alone fix this hypothesis upon a firm foundation. Such is the present state of the theory of comets, and such it must remain, till the patient observations and investigations of the practical Astronomer have established it upon a more certain basis.

(3.) Is there a first person singular in the ~~Infinitive~~ Mood?

By Mr. J. Hill, *Kilham Academy.*

It will not be difficult to shew, that the Imperative does not admit of a first person singular or plural; neither does it admit of more than one person. For the command, which it announces, is invariably addressed to the *second* person: for instance, in the verb *Let me love,* the address is evidently made to *thou* or *you,* and the construction is *Do thou,* or *do you, let me love.* Besides, if this Imperative admit of a first person, it must have a nominative case of the first person, whereas the first person *me* is the objective case following *let,* which has for its nominative *thou* or *you* understood: wherefore it will appear that *love,* in the above Example, is in the Infinitive mode, and that the Imperative is really entitled to one person only, and that is the *second;* for if it admit of no first person, it will admit of no third person; and if it admit of no first or third person in the singular, it will admit of neither in the plural number, because, whether we say *let me love* or *let him love,* whether we say *let us love* or *let them love,* the address is always the same.

(4.) Reasons for the apparently-increased magnitude of the sun and moon at the time of their rising and setting.

By Sir Daniel de Grandew.

The Phenomenon of the horizontal Moon has engaged the attention of some of the most eminent Philosophers in Europe.—Gassendus thought that, as the Moon was less bright in the Horizon, we looked at it there with a greater Pupil of the Eye, and therefore it appeared larger. This opinion was also supported by a French Abbe, who supposed that the opening of the Pupil made the chrystaline humor flatter, and the Eye longer, and by that means increased the Image. But this is contrary to the Principles of Optics; the image of an object upon the *retina* not depending

upon the Pupil. There is no connexion between the Muscles of the *Iris* and the other parts of the Eye, to produce this Effect.—M. R. I. Haüy observes, that we judge of Objects to be nearer or farther off accordingly as they are more or less clear, and our perception of them is more or less definite and distinct.—Also when between us and a remote object several other objects intervene, this new circumstance assists us still farther in estimating the Distance of the remote object, by a sort of addition which we make of all the distances of the intermediate objects, to form of them a total distance; from which it follows, that the object in this case appears to be farther off than it would if the intermediate space were unoccupied, and furnished us with no rallying point from which to compute the divisions of the distance.

To apply these principles to the Phenomenon in Question, the latter author observes that "we must reason from the Principle that we see the Sky in the form of a very depressed Vault, much less than a Hemisphere. This great concave expanse appears to us of a blue colour by reason of the different Strata of which the Atmosphere is composed, reflecting in preference the blue rays of the light of the Sun, and they are Rays passing to the Eye that occasion the Medium to appear to us of that colour." And it is easy to conceive, that a Spectator placed in the Centre of the Base of this depressed vault, (to whom the curvature of the Earth is imperceptible,) will perceive the extreme point of his sensible Horizon at a much greater distance than the Zenith, or any culminating point.—"On the other hand, the objects that are interposed between us and the Sun or Moon, when these luminaries are at the Horizon, contribute still farther to increase the the apparent Distance of those points (agreeably to the foregoing principle) with regard to the Spectator, and to diminish the curvature which he ascribes to the celestial Vault."

Now it is well known that the Disks of the Sun and Moon always appear under the same sensible angle, whether culminating or at the Horizon, as may be easily perceived by looking at them through a piece of smoked Glass, placed so near to the Eye as completely to eclipse all other objects, and afford no opportunity of estimating Distances, as was done by Malebranche. It is therefore evident that these luminaries, when at the Horizon, being referred to a greater Distance than when culminating, and the optic Angle remaining the same, we suppose them larger; for the products that result from the impression of the magnitude, combined with that of the distance having a common Quantity, (which is the first impression,) are in some sort proportional to the second; and thus we form an Idea of the real magnitude from the Proportion between the apparent distances.

Des Cartes, and our countrymen Wallis, Berkeley, Rowning, and Smith in his Optics, also latterly Dr. Okely, in Philosoph. Journal, No. 53. N. S., have given explanations of this Phenomenon in a way similar to the above. Dr. Smith makes the apparent distance in the horizon to be to that in the Zenith as 10 to 3, and therefore the apparent Diameters in that Ratio.

Mr. Ferguson observes, that we have two concurring appearances which deceive our imaginations and cause us to refer the Sun and Moon to a greater Distance at their rising or setting than when they are at a considerable height. *First*, their seeming to be on a more distant part of the Atmosphere; and, *Secondly*, their being seen through a grosser medium, which, by rendering them dimmer, causes us to imagine them to be at a yet greater distance. And as, in both cases, they are seen much under the same angle, we naturally judge them to be larger when they seem farthest from us.

In Vol. 64, of the Philosophical Transactions there is also a dissertation on this Subject by Dunn,

who was very attentive to the Phenomenon, and who, after making many judicious remarks about objects in the Horizon being enlarged while the Sun, just before his rising, has been passing apparently under them, and relating several Experiments about viewing objects through different Depths of Water &c. concludes thus, "The rays coming from the Sun, are, by the horizontal Vapours, first obstructed and many of them totally absorbed; the rest, proceeding with a retarded motion are thereby first reflected, and then less refracted through the humours of the Eye; and, lastly, that hereby the Image on the *Retina* becomes enlarged."

(5.) On the precedence of Theory or Practice in the business of teaching.

By Mr. B. Burnell, at the Free School, Batley.

It is a duty incumbent on every master to suit his instructions to the condition and capacity of his pupil. If it should be the intention of his parents to give him a good education, it would be better for him first to learn the theory. But, on the other hand, should his time for education be short and limited, it would be most advisable for him first to learn the practical part: For he who understands the practice, and not the theory, is, in my opinion, far better than he who knows the theory but not the practice.

We have many instances of boys who have learnt the theory perfectly well, who yet know nothing of the practice. It undoubtedly would be the best plan, for all masters to teach their pupils a little theory, and then the practice that belongs to it.

By Mr. John Baines, Nottingham.

The most natural Order of learning the Sciences, is to begin with the Theory; and then a solid Foundation will be laid, on which the Superstructure may be raised.

But it frequently takes up such a Length of Time this Way before any Thing useful can be attained, that it is perhaps advisable to begin with the Practice, especially where little Time can be allowed for the acquisition of scientific Knowledge. Besides, Youth have seldom any Taste for the Theory of any thing, before they see to what purposes it is applied; so that, when thy are compelled to learn it contrary to their Wills, they become disgusted with their Studies, and are addicted to Idleness. It is the great Art in the occupation of a Teacher, to make Learning delightful and easy, to smooth the apparently rugged Paths which lead to it, and by no means to discourage the Pupil by representing to him the agreeable Difficulties which he will have to encounter.

(6.) On the Quality and effects of Beer.

By INVESTIGATOR.

Very obvious is the reason of the difference between home-brewed beer, and that brewed by some publicans and brewers, in their respective quality and effects. When a man brews his own beer, he knows of what ingredients it consists. But when he confides in other people, he must take the liquor, of whatever composed, which they may be pleased to call *Beer*. Many publicans introduce into their mashing-tubs a decoction or extract of Quassia, Grains of Paradise, Coculus Indicus, Molasses, and various other ingredients of a similar taste and tendency. Although an Act of Parliament has long been in existence, prohibitory of all such admixtures, the mulct or penalty for which is excessively severe, yet there are adventurous souls who for the sake of gain will transgress all laws both of divine and human appointment. With such mortals, the deleterious effects of the articles which they sell or use, occupy no part of their consideration. The health of their fellow-men is with them of no consequence. Some of the greatest offenders in this respect are those per-

sons that in London are known by the name of "Brewers' Druggists," who open warehouses for vending by wholesale substitutes for hops and malt. Many of these men have acquired large fortunes by this nefarious traffic, always contriving to evade the Act of Parliament by selling their wares to persons that are not publicans or brewers.

In the calculation of crime, perhaps in some instances the country publican, or common brewer, is not so much to blame as are his drinking customers. It is a fact well ascertained that, in large towns, a hard-drinker will never frequent those inns or public-houses at which he finds ale or beer brewed entirely from malt and hops. His complaint invariably is, that "such a beverage does not feel *warm at heart;*" by which technical phrase, he endeavours to intimate, that it has no such cordial stimulating effect on his stomach, as that beer has which contains a strong decoction of Grains of Paradise, &c. To entice the regular professed drunkards, and for the sake of sparing a little expence, many publicans and brewers are foolish and wicked enough to prepare a liquor which shall convey a feeling of warmth to the heart and of stupefaction to the head. Those who are desirous of obtaining a salubrious and bracing drink, will avoid all such pest-houses, and will call for refreshment at those houses where they can meet with liquor that does not feel *warm at heart,* but which may nevertheless be intimately acquainted with Hop the Englishman and honest John Barleycorn.

(7) The cause of that extraordinary whirlpool called Maelstrom, on the coast of Norway.

By J. D. SOLLITT, *Holdgate, near York.*

The Island Moskoe, from whence the stream called the Maelstrom derives its name, lies between the mountain Hesleggen in Lefoden, and the island Ver, which are about one league distant; and between the island

and coast the stream makes its way. Between Moskoe and Lefoden, it is near four hundred fathoms deep, but between Moskoe and Ver, it is so shallow as not to afford passage for a small ship. When it is flood, the stream runs up the country, between Lefoden and Moskoe, with a boisterous rapidity; and, at ebb, returns to the sea, with a violence and noise unequalled by the loudest cataracts. It is heard at the distance of many leagues; and forms a vortex or whirlpool of great depth and extent: Its force and impetuosity are such that if a ship comes near it, being at once drawn irresistibly into the whirl, it there disappears, and is absorbed and carried down to the bottom in a moment, where it is dashed to pieces against the rocks: and just at the turn of ebb and flood, when the water becomes still for about a quarter of an hour, it rises again in scattered fragments, scarcely to be known for the parts of a ship. When it is agitated by a storm, it has reached vessels at the distance of more than a Norway mile, where the crews have thought themselves in perfect security. It was the opinion of Kircher that the Maelstrom is a sea-vortex, which attracts the flood under the shore of Norway, and discharges it again into the Gulph of Bothnia. But this opinion is now known to be erroneous, by the return of the shattered fragments of whatever happens to be sucked down by it. The large stems of firs and pines, rise again so shivered and splintered, that the pieces look as if covered with bristles. The whole phenomenon is the effect arising from the violence of the daily ebb and flow, occasioned by the contraction of the stream in its course between the rocks.

§†§ The gentlemen who have been accustomed to transmit to this work queries and answers to them, are desired in future to give their thoughts in writing on any subject to which their attention may have been particularly directed, or on which they may be wishful of eliciting the opinion of others; as it is not intended to propose queries in a formal manner in the next volume of this Miscellany. Literary men will not then be pinioned down to the discussion of a subject unsuited to their powers; but may give free scope to the bent of their genius.

ANSWERS

TO THE

QUESTIONS FOR YOUTH,

Proposed in the Third Number of the Second Volume.

(1) *By* Master Wm. Godward.

The product of three numbers in geometrical progression is 27, and the sum of their cubes is 757. What are those numbers?

Answered by Master John Roberts.

Let x be the first term and v the common ratio of the required progression. Then v, vx, v^2x will be the three numbers. Hence the Conditions of the Question give the following Equations $v^3x^3 = 27$, And $x^3 + v^3x^3 + v^6x^3 = 757$. Subtracting the former of these Equations from the latter gives $1 + v^6 \times x^3 = 750$. Hence by Substitution, we finally get $v^6 - \frac{750}{27}v^3 = 1$. The resolution of this quadratic gives $v = 3$. And consequently $x = \frac{3}{v} = 1$. And the three required Numbers are therefore 1, 3, 9.

The same by Master H. Emett.

If x, rx and r^2x be assumed for the terms of the progression, then by the conditions of the question we shall have $r^3x^3 = 27$ and $\left\{1 + r^3 + 2^6\right\} x^3 = 757$. The difference of these two equations gives us $\left\{1 + r^6\right\} x^3 = 730$. From the first equation $x^3 = 27 \div r^3$ substitute this instead of x^3 in $(1 + r)\, x^3 = 730$, and it be-

comes $(1 + r^6)\frac{27}{r^3} = 730$, or $27\,r^6 + 27 = 730\,r^3$, then $r^6 - \frac{730}{27}r^3 = -\frac{27}{27} = -1$. From the resolution of this equation we get $r = 3$: then $x = \sqrt[3]{\frac{27}{r^3}}\ \frac{3}{r} = \frac{3}{3}$ $= 1$, or the Progression required, is 1, 3, 9.

(2) *By* Master JOHN BOTTERILL.

Having lately to survey a field in form of a triangle, I observed that one of its angles was a right angle, and by the help of a cross found that the perpendicular fell 4 chains from one end of the hypothenuse and 3 from the other. Required the area of the field.

Answered by Masters W. H. CHRISTIAN AND G. GREGORY.

By a known property of the circle, $4 \times 3 = 12$ the square of the perpendicular, and $\sqrt{12} = 2\sqrt{3} =$ that perpendicular: hence the hypothenuse or base being equal to $4 + 3$ or 7, we shall have the area of the triangle equal to $7 \times \frac{2\sqrt{3}}{3} = 7\sqrt{3}$, or 12.1243556 square chains, equal to 1A. 0R. 34P. extremely near.

The same by Master RICHARD PEMBERTON.

Let $x =$ the base of the triangle: and having drawn the perpendicular by similar triangles $7 : x :: x : 4$, or $x^2 = 28$, and $49 - 28 = 21$ the square of the perpendicular $\therefore$ the Area of the triangle $= \frac{\sqrt{28} \times \sqrt{21}}{2} =$ $\sqrt{147} = 12.1243$ Chains $= 1$ acre, 34 perches.

Otherwise by Master THOMAS GASKIN.

Put $x =$ the perpendicular, then $x^2 + 16 =$ the square of one of the sides, and $x^2 + 9 =$ the square of the other, therefore $2x^2 + 25 = 49$, and by reduction

$x = \sqrt{12} = 3.4641016 \therefore \frac{3.4641016 \times 7}{2} = 12.1243$

556 Chains, the area required.

(3) *By* Master J. VINT.

Given $y^2 = z^2 + 10z + 100$, to find the values of y and z in positive whole numbers.

Answered by the Proposer.

Put $y^2 = z^2 + 10z + 100 = z^2 = 2nz + n^2$, then by transposition and division $z = \frac{n^2 - 100}{2n + 10}$ from whence $z = 6$, and $y = 14$ = two positive whole numbers.

Again by DURANDARTE.

Here $z^2 + 10z + 100$ must evidently be a complete square, the side of which put $= az - 10$. Hence $a^2z^2 - 20az + 100 = z^2 + 10z + 100$, or $a^2z - z = 20a + 10$. $\therefore z = \frac{20a + 10}{a^2 - 1}$; take $a = 4$: then $z = 6$, and $y = 14$.

Otherwise by Master C. E. COLLETT.

Put $y^2 = (z + a)^2 = z^2 + 2az + a^2 = z^2 + 10z + 100$, or $2az + a^2 = 10z + 100$ and by transposition $2az - 10z = 100 - a$, or $z = \frac{100 - a}{2a - 10}$ Now in order to get whole Numbers let $a = 8$, and then $z = 6$, and $y = 14$ = Answer.

(4) *By* Master WM. KITSON.

A circular grass plat 10 yards in diameter is to be increased by an area of an equal width going quite round it and containing just 50 square yards. Required the length of the string that strikes the circle.

Answered by Master J. GODWARD.

First $.7854 \times 10^2 = 78.54$ and $78.54 + 50 = 128.54$

area of the circle increased; then $\sqrt{\frac{128.54}{.7854}} = 12.7929$

then $\frac{12.7929}{2} = 6.39645$ length of the string.

Again by Master CHARLES WATERHOUSE.

Here $50 + (10^2 \times .7854) = 50 + 78.54 =$ area after being enlarged. Therefore $\sqrt{\left(\frac{128.54}{.7854}\right)} \div 2 = 6.39652 =$ length of the string.

(5) *By* Master JOHN STEPHENSON.

Given $x^4 + 2x^3 - x = 3080$ to find x by quadratics.

Answered by Master BLAKEY MARSHALL.

Here by completing the square $x^4 + 2x^3 - x + \frac{1}{4} = 3080.25$, Or $x^2 + x - \frac{1}{2} = 55.5$ by Evolution; Or $x^2 + x = 56$. Then $x^2 + x + \frac{1}{4} = 56.25$, by completing the square; consequently $x + \frac{1}{2} = 7.5$; Or $x = 7$.

Again by Master JOHN SIMPSON.

First by transposition $x^4 + 2x^3 - x - 3080 = 0$, or $(x^2 + x - 56) \times (x^2 + x + 55) = 0$. Now in order to have a positive root for x, Let us make $x^2 + x - 56 = 0$: and by completing the square and extracting the root we find $x = 7$.

Otherwise by Master B. BURNELL.

By multiplication, square root, &c. we get $2x^3 + 2x^2 = 112x$, and by division $x^2 + x = 56$, then $x + x + .25 = 56 + .25 = 56.25$, and $x + .5 = \sqrt{56.25} = 7.5$; hence $x = 7.5 - .5 = 7$.

Answered by Master JASON BARROWCLOUGH.

Now to take away the second term let $x = z - \frac{1}{2}$

Then $\left\{\begin{array}{l} x^4 = z^4 - 2z^3 + \frac{3}{2}z^2 - \frac{1}{2}z + \frac{1}{16} \\ 2x^3 = \quad 2z^3 - 3z^2 + \frac{3}{2}z - \frac{1}{4} \\ -x = \quad -z + \frac{1}{2} \end{array}\right\} 3080$

Whence by addition $z^4 - \frac{3}{8} z^2 = \frac{49275}{16}$, this quad, solved $z = \sqrt{\frac{(12321)\frac{1}{2}}{4} + \frac{3}{4}}$ 7.5 hence $x = 7$.

(6) *By* Miss MARY WEST.

What difference of time will water be in descending along a drain of 3 miles in length and 3 feet fall, and another of 20 miles in length and 40 feet fall?

Answered by CHARLES SAUNDERS AND RICHARD WEST.

$\sqrt{3 \div 16\frac{1}{12}} = \sqrt{\frac{36}{193}}$ Seconds, time of descent down a perpendicular of 3 feet fall. Then $3 ft. : \sqrt{\frac{36 sec.}{193}}$ :: 3 *miles* : 38′ 0″.37 time of descent down the former drain. And $\sqrt{\left(40 \div 16\frac{1}{12}\right)} = \sqrt{\frac{480}{193}}$ seconds, time o descent down a perpendicular of 40*ft.* fall. $40 ft : \sqrt{\frac{480}{193}}$ :: 20 *miles* : 69′.23″.37 time of descent down the latter drain. ∴ 69′ 23″.37 — 38′ 0″.37 = 31′ 23″ the difference of time required.

(7) *By* Master T. BURLEY.

Three sides of a trapezium are 420, 496, and 513 links, and the angle included between the first and second sides is = 35° 25′ and that between the third and fourth = 27° 19′. Required the fourth side and the area of the trapezium.

Answered by Master JOSEPH GREENHOUGH.

The natural sine f 35° 25′ is .5795183, and (.5795183 × 420 × 496) ÷ 2 = 60362.626128; and, by trigonometry, I find the fourth side to be 621.454. Then the natural sine of 27° 19′ is .4589080; and (.4589080 × 513 × 621.454) ÷ 2 = 73151.289437508 0. then 73

131. 2894375080 + 60662. 626128 = 133513. 915565 5080 = 1*a*. 1*r*. 13*p*. the area of the trapezium.

(8) *By* Master THOMAS SAMPSON.

If the diameter of a globe of marble be 125 inches, what is the solidity of a segment sawn off it, that cost 12*£*. 5*s*. 0½*d*. polishing at a farthing per square inch?

Answered by Master CHARLES THOMAS.

Let $d = 125$, $s = 11762$ = the convex surface of the segment, $x =$ its height, and $p = 3.14159$; then by the question $pdx = s$, or $x = \frac{s}{pd}$; and $(3d - 2x) \times x^2 \times \frac{p}{6} = \left(\frac{dp}{2} - \frac{s}{3d}\right) \times \frac{s^2}{p^2d^2} = 148007.635$ inches, the area.

Ingenious answers to all the questions were given by Masters J. Roberts, *Mr. Swale's Academy, Liverpool;* Thomas Gaskin, *Mr. Jaques's School, Penrith;* C. E. Collett, *Mr. Fletcher's Academy, Armley;* Durandarte, *Barningham;* Joseph Greenhough, *Bolton near Bradford;* Charles Waterhouse, *Mr. Garside's Academy, Mirfield;* Charles Saunders, *Mr. Newby's Academy, Barningham;* C. Thomas, *Eccleshill;* Richard Pemberton, *Bingley;* W. Godward, *Free Writing School, Sheffield;* Blakey Marshall, *Mr Garside's Academy, Mirfield.*

John Hodge, *Egg-Buckland;* Jason Barrowlough, *Sowerby near Halifax,* answered 7 questions.

Miss Mary Jackson, *Gatherleen near Gainsborough,* answered 6 questions.

John Simpson, *Armley,* John Godward, *Free Writing School, Sheffield,* James A. Harvey, *Grammar School, Kington, Herefordshire,* G. Gregory, *Tamerton Folliott, near Plymouth,* answered 5 questions.

W. H. Christian, *Mr Swale's Academy, Liverpool,* H. Emett, *Mr. Swale's Academy, Liverpool,* Matthew Sanderson, *Mr. Jaques's School, Penrith,* answered 4 questions.

ANSWERS

TO THE

MATHEMATICAL QUESTIONS

Proposed in the Third Number of the Second Volume.

(143) *By* Mr. JOHN BAINES, Jun.

Find three numbers in arithmetical progression such than n times the sum of every two of them may be a square number.

Answered by MR. E. WEBSTER, *Armley Mills.*

Suppose $x-y, x$, and $x+y$ to be the required numbers. Then per question $\left\{\begin{array}{l}(2x+y)\times n = p^2 n^2\\(2x-y)\times n = q^2 n^2\\2x \times n = r^2 n^2\end{array}\right\}$ Hence by division and substitution we have $\left\{\begin{array}{l}r^2n + y =\\r^2n - y =\end{array}\right.$ $\left.\begin{array}{l}p^2 n\\q^2 n\end{array}\right\}$ therefore by equating these two we find $y = \frac{p^2 - q^2}{2} \times n$, and $r^2 = \frac{p^2+q^2}{2}$. Now it is evident $\frac{p^2+q^2}{2}$, or $2p^2 + 2q^2$ must become a square. In order to do this make $p+q=m$, and $p-q=t$, then our form becomes $m^2 + t^2 = \square$. Consequently if $m = u^2 - v^2$, and $t = 2uv$ the question is solved, where u and v may be any numbers at pleasure. If $u=3$, and $v=1$ $\therefore p=7$ and $q=1$. Wherefore $y=24n$, and $x = \frac{25n}{2}$ Let $n=2$, then $x=25$, and $y=48$, numbers that answer the question.

The same by MR. JOHN BAINES. Jun.

Let $\frac{n}{2}.(x^2 + y^2) - 2\,n\,x\,y$, $\frac{n}{2}.(x^2+y^2)$, and $\frac{n}{2}.(x^2 + y^2) + 2\,n\,x\,y$ represent the three numbers in arithmetical progression; then n times the sum of every two of them will be $n^2.(x^2 - 2\,x\,y + y^2)$, $n^2.(x^2 + y^2)$, and $n^2.(x^2 + 2\,x\,y + y^2)$ to be made squares; but the first and last are squares, and it only remains to make $n^2.(x^2 + y^2)$ or $x^2 + y^2$ a square, which is known to be so when $x = r^2 - s^2$, and $y = 2\,r\,s$; hence the three required numbers are $\frac{n}{2}.(r^2 + s^2)^2 - 4\,n\,r\,s.(r^2 - s^2)$, $\frac{n}{2}.(r^2 + s^2)^2$, and $\frac{n}{2}.(r^2 + s^2)^2 + 4\,n\,r\,s.(r^2 - s^2)$ where r and s may be taken at pleasure, provided r be equal to, or greater than $7\frac{1}{2}\,s$, so that the first number may be positive. Example, suppose $r = 9$, $s = 1$, and let n be given $= 3$, then we obtain 1446,10086, and 18726 for the three numbers required, &c. &c.

Ingenious answers were also given by Messrs. Maffett, S. H. F. Whitehead, Garside, Johnson, Drake, Sollitt, Lewthwaite, and Ambulator.

(144) *By* Mr. B. C. GOODISON, *at the Rev. H. Davis' Seminary, Eaton Bishop, near Hereford.*

Given the hypothenuse of a right angled triangle $= 8$, to determine it when the three sides are in harmonical ratio.

Answered by MR. JOSHUA GARSIDE, *Mirfield.*

Put $8 = a$, and let $x =$ the perpendicular of the triangle, then will the base $= \sqrt{(a^2 - x^2)}$. Now, by the nature of harmonical proportion, $a - \sqrt{(a^2 - x^2)} : \sqrt{(a^2 - x^2)} - x :: a : x$, then by multiplying extremes and means we get $a\,x - x\,\sqrt{(a^2 - x^2)} = a\,\sqrt{(a^2 -$

$x^2) - a\,x$, or $2\,a\,x = a + x.\ \sqrt{(a^2 - x^2)}$; but by division $\frac{2\,a\,x}{a+x} = \sqrt{(a^2 - x^2)}$, or $\left(\frac{2\,a\,x}{a+x}\right)^2 = a^2 - x^2$. Hence by reduction we get $x^4 + 2\,a\,x^3 + 4\,a^2\,x^2 - 16\,a^2\,x - 64\,a^2 = 0$. Then by the method of trial and error $x = 5.058339$, the perpendicular, and the base $\sqrt{(a^2 - x^2)} = 6.1978388$. So that 8, 6.1978388, and 5.058339, are the sides of the triangle, which are in harmonical proportion, as required.

Again by MR. JOHN BAINES, Jun. *Nottingham, and* MR. ROBERT MAFFETT, *Plymouth.*

Put $a = 8 =$ the hypothenuse, and $x =$ the base of the required right-angled triangle; then $\sqrt{(a^2 - x^2)} =$ the perpendicular. Now a, $\sqrt{(a^2 - x^2)}$ and x are to be in harmonical ratio; but when three quantities are in harmonical ratio, their reciprocals are in arithmetical progression, and therefore $\frac{1}{a} + \frac{1}{x} = \frac{2}{\sqrt{(a^2 - x^2)}}$ or $x^4 + 2\,a\,x^3 + 4\,a^2\,x^2 - 2\,a^3\,x - a^4 = 0$, from which by approximation we find $x = .6322924\,a$; hence $\sqrt{(a^2 - x^2)} = .7747298\,a$, and the triangle becomes known.

N. B. The above equation has only one positive root; the other three are, two imaginary and one negative; therefore the question admits only of one rational solution.

Otherwise by MR. T. DAVIES, *Sheffield.*

Let s and c represent the sines of the acute angles, then per Trig: the opposite sides are $8s$, and $8c$; and by the nature of harmonical proportion, $8 : 8c :: 8 - 8s : 8s - 8c$, and $c - sc = s - c$, or $c = \frac{s}{2 - s}$ from which $s = .77473$, and $c = .63229$; these multiplied by 8 give 6.19784 and 5.05832 the numbers required.

Thus nearly is the answer given by Messrs. Shenstone, Postlethwaite, West, S. H. F. Whitehead, Johnson, Webster, Drake, Sollitt, Lewthwaite, and Ambulator.

(145) *By* T——*n, Wakefield.*

Required the roots of the equation $x^4 - 4\frac{1}{2}\sqrt{2}x^3 + 6\frac{1}{2}\sqrt{4}x^2 - 4\frac{1}{2}\sqrt{8}x + 2 = 0$.

Answered by Mr. JOHN D. SOLLITT, *Teacher of the Mathematics, Holdgate, near York.*

It is evident from inspection that this equation is $= (x - \sqrt[4]{2})^4$, **therefore its roots without any further trouble are** $= \pm \sqrt[4]{2}$.

Again by MR. JAMES WHITEHEAD,* *at Mr Garside's Academy, Mirfield.*

First $x^4 - 4x^3\sqrt[4]{2} + 6x^2\sqrt{4} - 4x\sqrt[4]{8} + 2 = (x - \sqrt[4]{2})^4 = 0$; **hence** $x - \sqrt[4]{2} = 0$, **or** $x = \sqrt[4]{2}$, **therefore the equation has 4 equal roots.**

True and ingenious solutions are also given by Messrs. Garside, Johnson, Baines, Webster, Drake, Lewthwaite, and Ambulator.

(146) *By* Mr. J. B. HAWKINS, *at Mr. Smith's Academy, York.*

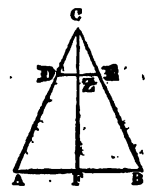
Given the altitude of a candlestick having a circular top, and standing upon a horizontal table, to determine the height of the apex of the flame of a candle placed vertically therein, when the area of the top is to that of its shadow upon the table in any given ratio.

Answered by Mr. E. WEBSTER and Mr. J. BAINES, *whose solutions are almost identical.*

Suppose FZ to represent the candlestick, DE the diameter of the circular top, and AB the diameter of its

* This young man wishes to engage as mathematical assistant in some respectable seminary.

shadow, also ZC the height of the flame above the candlestick and let the given ratio be as $m : n$, then by the nature of the question $AB^2 : DE^2 : m : n$; but by similar triangles $AB : DE :: FC : CZ :: \sqrt{m} : \sqrt{n}$. Hence $CZ = \frac{\sqrt{n}}{\sqrt{m} - \sqrt{n}} \times FZ$.



Answered by Messrs. Shenstone, Postlethwaite, Maffett, England, S. H. F. Whitehead, Garside, Johnson, Drake, Sollitt, Lemkwaite, Ambulator, and Hawkins.

(147) *By* Mr. Wm. Wisheman, *Hull.*

Required the content in wine gallons of the solid formed by the revolution of the complete cissoid about its axis (30 inches).

Answered by Mr. J. D. Sollitt, and Mr. J. Baines.

If d represent the diameter of the generating semicircle, the equation of the cissoid will be $y^2 = \frac{x^3}{d - x}$ (Hutton's Course, Vol. III, p. 205,) and the solidity of the solid generated by it round its axis, $p\, d^5 \times h.\, l.\, \frac{d}{d - x} - (\frac{1}{5} p\, x^5 + \frac{1}{2} p\, d\, x^4 + p\, d^2 x)$, (Simpson's Fluxions, Davis's Edition, page 127,) which when $x = \frac{2}{3} d$, or the cissoid is complete, becomes $= p.\ (d^5 \times h.\, l.\, 2 - (\frac{2}{5} d^5)) = .0264806\ p\ d^5 = .083191452\ d^5$, which when $d = 30$ inches, becomes 2246.1692 cubic inches $= 9.72367$ wine gallons.

Again by Mr. Elias Webster, and Mr. Joshua Garside.

By Simpson's Fluxions, Vol. 1st, Art. 150, The solidity of the cissoid whose diameter $= a$, abscise $= x$, and

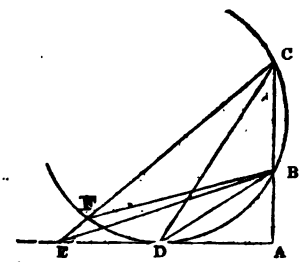
$p = 3.1416$, is $p \times -\frac{x^3}{3} - \frac{a x^2}{2} - a^2 x + a^3 \times$ hyp. log of $\frac{a}{a - x}$; Therefore when $x = 15$, and $a = 30$, (as per question being complete in this case,) the solidity will be 2246, 16277295 or $9\ \frac{2}{11}$ gallons nearly as required.

Ingenious solutions to this question were also received from Messrs. Whitehead, Johnson, Baines, Webster, Drake, Sollitt, and Lewthwaite.

(148) *By* Mr. E. T. THORNE.

A flagstaff of given height is erected on a tower whose height is also given: at what point on the horizon will the flagstaff appear under the greatest possible $\angle$.

Answered by GULIELMUS GIBSONI, *Normanby, Lincolnshire.*



Let AB = the height of the tower. BC = height of the staff, draw AE perpendicular to AB, and make AD = $\sqrt{(AB \times AC)}$, and AD will be the distance required. For, through the points C, B, D, describe a circle, and join DB, DC, and take any other point E in the horizontal line AE, and join EC cutting the circle in F; also join EB, FB. Now the $\angle$ CDB = $\angle$ CFB, being in the same segment of a circle. But (Euc. 1, 16.) CFB or CDB is greater than the $\angle$ CEB.

The same may be demonstrated of any other point on either side of the point D; for the circle touches the line AE in the point D only, (Euc. 3. 36.) and AC : AD :: AD : AB.

Again by Mr. WEBSTER and Mr. S. H. FLETCHER, *Armley, near Leeds.*

In the preceding figure, AD represents the horizon, AB the tower, and CB the flagstaff. Then (by Simpson's Select Exercises, problem 48,) the angle CDB will be a maximum, when a circle passing through the two points C and B touches the line AD, or when AD is a mean proportional between BA and AC.

Otherwise by the proposer, E. T.

Suppose BA the tower, CB the flagstaff, D a point in the horizon. Join CD, DB. Put A C $= a$, BA $= b$, and CAD $= x$; suppose rad. $= 1$, then $x : b :: 1 : \frac{b}{x} =$ tangent $\angle$ ADB; and $x : a :: 1 : \frac{a}{x}$ tangent $\angle$ CDA

$\therefore \frac{\frac{a-b}{x}}{1+\frac{ab}{x^2}} =$ T. of their difference, which is a maximum, and which put into fluxions and reduced gives $ab - x^2 = 0 \therefore a : x :: x : b$; consequently the point on the horizontal plane DA is determined.

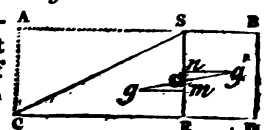
Answered by Messrs. Shenstone, Postlethwaite, Treeby, England, Whitehead, Garside, Johnson, Baines, Thorne, Drake, Sollitt, Lewthwaite, Ambulator, and Maffett.

(149) *By the same.*

Having given a rectangular piece of metal of uniform thickness, it is required from one of its angles to draw a straight line cutting off a triangle in such a manner, that the remaining trapezium being suspended by the obtuse angle the parallel sides of the trapezium may remain horizontal.

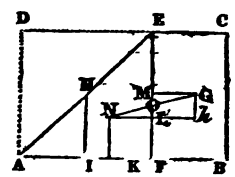
Answered by Mr. JOSEPH LEWTHWAITE, *Master of the National School, Halifax.*

Let SBDC represent the remaining trapezium, S the point of suspension, G its centre of gravity, which, according to the conditions of the question, must fall in the line SE parallel to BD. Also, let g, $'g$, be the centres of gravity, of the triangle SEC, and rectangle SBDE, and draw the perpendiculars gm, $'gn$. Then by Mechanics, SEC $\times gm =$ SBDE, $\times 'gn$, or SE (BD) $\times \frac{1}{2}$ CE $\times \frac{1}{3}$ CE $=$ BD $\times$ DE $\times \frac{1}{2}$ DE, or $\frac{1}{6}$ CE$^2 = \frac{1}{2}$ DE^2 or CE $= \sqrt{3} \times$ DE. Hence $1 : \sqrt{3} ::$ DE : CE, and by comp. $1 + \sqrt{3}$: CD :: 1 : DE $=$ BS.



Again by GULIELMUS GIBSONI.

Let ABCD represent the rectangular piece of metal, E the point of suspension, ADE the piece cut off. Also let N be the centre of gravity of the $\triangle$AFE, and G the centre of gravity of FBCE. Join NG, and since the sides AB, CE, are by the question parallel to the Horizon, the centre of gravity of ABCE will be in the line EF, $\perp$ to AB and conseq. in O where it intersects NG. Now if we consider NG as a lever, and O its fulcrum, and suppose the masses on each side collected in their repective centres of gravity N and G, and put AB $= a$, BC $= b$, FB $=$ EC $= x$, then will AF $= (a - x.)$



And in case of equilibrium $\frac{\text{AF} \times \text{FE} \times \text{LN}}{2} =$ FB $\times$ FE $\times$ MG. But FN $= \frac{2}{3}$ FH as is well known. $\because$ (by sim. $\triangle$s. FHI, FNK), FK $=$ LN $= \frac{2}{3}$ FI $= \frac{1}{3}$ AF, or $\frac{1}{3}(a - x)$, and MG $= \frac{1}{2}$ FB $= \frac{1}{2}x$. Wherefore, by rejecting the common quantity FE in the above equation and substituting for the remaining

quantities their respective values we shall have $\frac{a-x}{2} \times \frac{a-x}{3} = x \times \frac{1}{2}x$, or when ordered $x^2 + ax = \frac{a^2}{2}$; which reduced, gives $x = \pm\sqrt{\frac{3a^2}{4}} - \frac{a}{2} = \pm\frac{a}{2}\sqrt{3} - \frac{a}{2}$.

Otherwise by Mr. F. DRAKE, *at Mr. Garside's Academy, Mirfield.*

Put the length of the rectangle CD $= a$, and its breadth AD $= b$, and $n =$ DE; then will $a - n =$ CE and the centre of gravity of the triangle AEF will be N, NL being $\frac{1}{3}n$. Also the centre of gravity of the rectangle FC is G. Hence if $r =$ GN we shall have $\frac{2ab - bn}{2} : \frac{bn}{2} :: r : \frac{rn}{2a-n} =$ OG; and $r - \frac{rn}{2a-n} = \frac{2ar - 2rn}{2a - n} =$ ON. Then since NL $= \frac{n}{3}$, and L$h = \frac{a-n}{2}$: we have $\frac{3a - n}{6} =$ Nh; whence by similar triangles $r : \frac{3a-n}{6} :: \frac{2ar - 2rn}{2a - n} : \frac{3a^2 - 4an + n^2}{6a - 3n} =$ NL $= \frac{n}{3}$; whence $n^2 - 3an = -\frac{3a^2}{2}$: then by completing the square &c. we have $n - \frac{3a}{2} = \pm\frac{a\sqrt{3}}{2}$ and taking $\frac{a\sqrt{3}}{2}$ negative, $n = \frac{3a - a\sqrt{3}}{2}$: or $n = .6339746\,a =$ BDEC as required.

It was likewise answered by Messrs. Baines, Garside, Shenstone Postlethwaite, England, Treeby, Whitehead, Johnson, Webster, Sollitt, and Ambulator.

(150) *By* Mr. Joshua Garside, *Mirfield.*

Suppose that 9 planets perform their respective revolutions round their sun in 1, 2, 5, 7, 8, 12, 15, 16, and 21 years of our time, and that they were all in exact conjunction in the sign libra at the end of the year A.D. 137. I desire to know in what year of Christ the next grand conjunction would happen.

Answered by Mr. Joshua Garside *the proposer, and* Mr. S. H. Fletcher, *Armley.*

Put n for the number of years, that will elapse between the given conjunction and the next that will happen, then if n is the multiple of 12, 15, 16, and 21 it is also the multiple of 1, 2, 5, 7, and 8, therefore $\frac{n}{12}$, $\frac{n}{15}$, $\frac{n}{16}$, and $\frac{n}{21}$, must be integers, put $\frac{n}{12} = p$, then $n = 12p$ which substituted in the second fraction, gives $\frac{12p}{15}$, or $\frac{4p}{5} = wh$, but $\frac{5p}{5} = wh$, then $\frac{5p}{5} - \frac{4p}{5} = \frac{p}{5} = wh = r$, then $p = 5r$, and $n = 60r$, and this substituted in the third fraction gives $\frac{60r}{16} = 3 + \frac{3r}{4} = wh$, but $\frac{3r}{4} = wh$, and also $\frac{4r}{4} = wh$, then $\frac{4r}{4} - \frac{3r}{4} = \frac{r}{4} = wh = s$, then $r = 4s$, and $n = 240s$, which substituted in the last fraction gives $\frac{240s}{21} = 11 + \frac{3s}{7} = wh$, but $\frac{3s}{7} = wh$, and $\frac{3s}{7} \times 2$

$= \frac{6s}{7} = wh$, also $\frac{7s}{7} = wh$, then $\frac{7s}{7} - \frac{6s}{7} = \frac{s}{7} = wh$, $= t$, or $s = 7t$; consequently $n = 1680s$, and if $s = 1$, then $n = 1680$, and $1680 + 137 = 1817$, the year of Christ required.

Again by Mr. J. D. Sollitt.

It is evident that the least whole number divisible by 12, 15, 16 and 21, will answer the conditions of the question, put $x =$ the number, and $\frac{x}{12} = p$, or $x = 12p$, hence $\frac{12p}{15} = wh$, or $\frac{4p}{5} = wh$, or $\frac{p}{5} = q$, hence $p = 5q$, or $60q = x$, therefore $\frac{60q}{16} = wh$, or $\frac{3q}{4} = wh$. hence $\frac{q}{4} = r$, or $240r = x$, and $\frac{240r}{21} = wh$, therefore $\frac{3r}{7} = wh$, or $\frac{r}{7} = s$, and if $s = 1$, $x = 240 \times 7 = 1680$; and $1680 + 137 = 1817$ the required year.

Otherwise by Mr. John Baines, Jun.

This question is evidently no more than to find the least common multiple of the given numbers, 1, 2, 5, 7, 8, 12, 15, 16, and 21, which may be expeditiously performed by the rule given in Keith's Arithmetic page 76, 6*th* edition. In the present case, the divisors will be found to be 2, 2, 2, 3, 5, and 7, and the quotients (omitting those which are unity) 2; hence $2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 2 = 1680$ years before the planets will come together, consequently the next grand conjunction would be in the year $(137 + 1680 =)$ 1817.

Note. When the Times of revolution consist of several denominations, the question may be solved in the same manner, after the times have all been reduced to the lowest name mentioned.

Again also by Ambulator, *Canterbury.*

Nothing more is required to answer this question

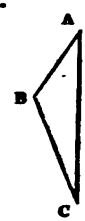
than to find a common multiple of 1, 2, 5, 7, 8, 12, 15, 16, 21 years. Now 70 is a multiple of 1, 2, 5, 7, because these numbers are prime to each other. And $\frac{70 \times 8}{2} = 280$, (Wood's Alg. Art. 374): by proceeding in the same manner with the remaining numbers, we obtain 1680 the common multiple sought, therefore $1680 + 137 = 1817$, the required year.

Answers were received from Messrs. Webster, West, S. H. F. Whitehead, Johnson, and Lewthwaite.

(151) *By* Job Shainer.

On the 15th of July, 1816, in lat 53° 45′ N. at 36′ minutes past 9 o'clock in the morning, I observed a small cloud exactly in the east, the shadow of which fell upon the earth at the distance of 1270 yards, and in the direction E.N.E. from the place where I stood. Required the perpendicular height of the cloud.

Answered by Messrs. J. Baines, J. Shenstone, R. West, and J. Postlethwaite.



By the Ephemeris, the Sun's declination at the given time was 21° 33′ N.; also $12h. - 9h. 36' = 2h. 24' = 36°$, the Sun's angular distance from noon. Hence there are given the Co-lat. = 36° 15′, the Co-declin. = 68° 27′, and the hour angle 36°, to find the Sun's Alt. 47° 50′, and the Azimuth 125° 28½′ from the north. Let A (in the annexed figure) represent the place of observation, B the spot where the shadow fell, and C the spot perpendicularly below the cloud; then AB is given = 1270 yards, the $\angle ACB = 35° 28\frac{1}{2}'$ the Azimuth from the East, and the $\angle CAB = 22° 30'$ = the E.N.E. point of the compass from the East, to find the side BC = 837.44 yards, the horizontal distance of the shadow from the point perpen-

dicularly below the cloud. Hence, to find the perpendicular height of the cloud, we have given the base 337.44 yards, and the angle at the base 47° 50′ (= the Sun's Alt.) in a right angled triangle, to find the perpendicular = 924.64 yards, the perpendicular height of the cloud.

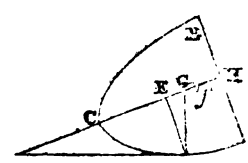
N.B. No allowance is made in the above solution for the Semi-diameter, Refraction, or Parallax.

It was also ingeniously answered by Messrs. West, Whitehead, Garside, Johnson, Webster, Ambulator, S.H.F., Sollitt, and Lewthwaite.

(152) *By* Mr. BAINES.

If a paraboloid whose diameter is 8 and altitude $10\frac{1}{2}$ be rolled on a smooth horizontal plane, in such a manner that the axis produced may always pass through the same point: it is required to determine the area of the curve generated by the point of contact.

Answered by Messrs. AMBULATOR, BAINES, GARSIDE, J. D. SOLLITT, and JAMES POSTLETHWAITE.



Let ADCB represent the Paraboloid, AB its base, HC its axis, G its centre of gravity, GD the shortest distance from thence to the curve, and DF a tangent to the parabola at that point, meeting the axis produced in F. Draw DE parallel to AH. Because the Paraboloid will touch the plane at D, the radius of the circle described by the point of contact will be equal to the tangent DF. Put EG = x; then because CG = $\frac{2}{3}$ CH, CE = $7 - x$, and by the property of the Parabola CH : AH^2 :: CE : DE^2 = $\frac{224 - 32x}{21}$; but DE^2 + EG^2 = DG^2 = $\frac{21x^2 - 32x + 224}{21}$, and this by the

nature of the question must be a minimum, or $21x^2 - 32x$ = a min. which in fluxions is $42xx - 32\dot{x} = 0$, therefore $x = \frac{16}{21}$, hence $CE = 7 - x = 6\frac{5}{21}$. and $DE^2 = \frac{4192}{441}$; but by the property of the parabolic tangent $CF = CE$, $\therefore EF = 2CE = 12\frac{10}{21}$, and $\sqrt{EF^2 + DE^2} = DF = \frac{8}{21}\sqrt{18209}$ = the radius of the circle, consequently $.7854 \times 4 \times (\frac{8}{21}\sqrt{18209})^2 = 3.1416 \times \frac{4}{441} \times 18209 = 518.86979$ = its area required.

Answered by Messrs. J. LEWTHWAITE, and ELIAS WEBSTER.

Let ABC in the preceding Figure represent a section of the given paraboloid, touching the horizontal line FD in D; G its centre of Gravity situated perpendicularly over D; also, let DE be drawn perpendicular to FH the axis produced. Then by the writers on fluxions $CG = \frac{2}{3}CH$ and by the nature of the parabola $GE = \frac{BH^2}{2CH} = \frac{16}{21}$, $CE = CG - EG = 7 - \frac{16}{21} = 6\frac{5}{21}$, $FE = 2CE = 12\frac{10}{21}$, and $FG = FE + EG = 13\frac{5}{21}$. Then per sim. $\triangle$'s FE : FD :: FD : FG, and $DF^2 = GF \times FE = 13\frac{5}{21} \times 12\frac{10}{21}$, this multiplied by 3.1416 gives 518.86979, the area of the circle required.

It was likewise answered by Messrs. Shenstone, Whitehead, and Johnson.

(153) *By* Mr. E. WEBSTER, *Armley Mills.*

Find three numbers such that the sum of every two of them may be a biquadrate, and the sum of all three a fourth power and a cube.

Answered by Mr. E. WEBSTER, *Armley.*

Let $x = \frac{n^4 + v^4 - s^4}{2}$, $y = \frac{s^4 + v^4 - n^4}{2}$, and $z = \frac{n^4 + s^4 - v^4}{2}$, then the first three conditions are answered. Secondly make $n = v - s$, then $x = v^4 - 2v^3s + 3v^2s^2 - 2s^3v$, $y = 2v^3s - 3v^2s^2 + 2s^3v$, and $z = s^4 - 2s^3v + 3v^2s^2 - 2v^3s$. Therefore $x + y + z = (v^2 - vs + s^2)^2$. Now in order to make this a biquadrate we must make $v^2 - vs + s^2$ a square, which is the case when $v = (t^2 - 1) \times m$, and $s = (2t + 1) \times m$; moreover we can make this form a cube also, for $v^2 - vs + s^2 = (t^2 - t + 1)^2 \times m^2$ which may be made equal to $m^3 p^3$; hence $m = \frac{(t^2 - t + 1)^2}{p^3}$. Let $p = 1$ and $t = 3$, then $v = 392$, $s = 245$, and $n = 147$ Cons. $x = 2320 \times (49)^4$, $y = 1776 \times (49)^4$ and $z = -1695 \times (49)^4$.

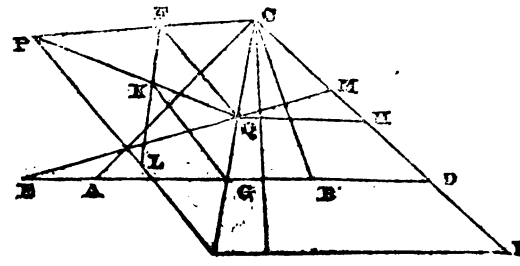
True and ingenious solutions are also given by Messrs. Johnson, and Baines.

(154) *By* AMBULATOR, *of Canterbury.*

Given the perpendicular, the radius of the inscribed circle, and the straight line bisecting the vertical angle and terminating in the circumference of the circumscribing circle, to construct the plane triangle.

Answered by Mr. J. H. SWALE, *Liverpool.*

In this question, instead of the perpendicular and radius of the inscribed circle, let there be given two lines drawn from the vertex of the △ and the centre of the inscribed circle, to make any given ∠s with the base, and the remaining datum as per question. With these data, suppose the △ ACB constituted; CD, QE, being the given lines drawn from C the vertex, and Q, the centre of the inscribed circle, making the given △s CDG, QEG, with the base AB: and in CG produced, take CI equal the given bisecting line; draw QH par. to AB, and produce EQ, to meet CD at M. Because



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of given ∠s and pars. QE : HD = MQ : MH is a given ratio, and QE a given line, or DH, HC, and the ratio CQ : QG = CH : HD are given: also, by a well known property of △s, IC : IQ = IQ : IG: We have then, the line DC or the point C and the indefinite parallels DG, HQ, given in position, to draw CQGI so, that CI shall be a given line and CI. IG = IG². Or which is the same thing, to determine the points Q, G, in the given right line CI, making CQ : QG a given ratio, and CI. IG = IQ². The problem in the latter form, admits of the following simple

Comp. From the extremities C, I, of the given line, draw CP, IP, to any point, P, make PC : PF = the given ratio, draw FQ, FL, par. to PI, CI, join PQ, cutting FL in K, then KG being drawn par. to FQ, it is done. For per parallels, CQ : FK = CQ : QG = PC : PF the given ratio, and IC : IQ = LF : LK = IQ : IG, or CI. IG = IQ² : the rest is obvious.

The problem of drawing CQGI to meet the parallels HQ, DG, making CI. IG = IQ², would be easily effected, by producing CD, so that CR. RD = RH², an easy problem, drawing RI par. to DG, and applying to RI, the line CI of the given length. This mode of determination gives two positions of I or of CI, equally answering the conditions of the question: Also we hence deduce a solution for the case, where CI, the bi-

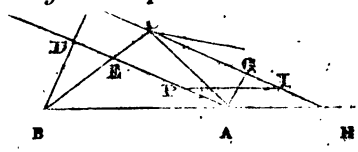
secting line, is a minimum. For, having drawn the parallel RI as above, draw CI perpendicular thereto, which determines I, and consequently the △ ACB, for the minimum: the maximum being infinite.

Answers were received from Messrs. Postlethwaite, Maffett, Shenstone, Treeby, S. H. F., Johnson, Webster, and Sollitt.

(155) *By* Mr. PETER DICKENEER, *Peterborough.*

If AB the base of a plane triangle ACB be given, and the side BC cut the straight line AD given in position in E, so that the triangle AEC may be always equal to a given space: what is the locus of the vertex C?

Answered by the Proposer.



Draw CH parallel and BD, AG perpendicular to AD, which divide in P so that the rectangle BD × AP shall be equal to twice the given area of the △ AEC; then AP will be a given line, P a given point, and PI drawn parallel to AB, will be given by position. Now the triangles BCH, BEA as also the triangles BDA, AGH are similar: hence

BH : CH :: AB : AE
AB : BD :: AH : AG.

Whence comp. BH : CH . BD :: AH : AE . AG = BD . AP. Therefore BH : CH : AH :: AP = IH
Alternately BH : AH :: CH : IH
Dividendo AB : AH = PI :: CI : IH = AP
Therefore PI × CI = AB × AP, a given rectangle: consequently the *locus* of the point C is a given *hyperbola* whose asymptotes are PD and PI.

*** Mr. Samuel Treeby and Mr. John D. Sollitt both determine analytically the Locus of the vertex of the triangle to be a hyperbola; but one of our correspondents makes it to move in a right line given in position, whilst another asserts that the same locus is a

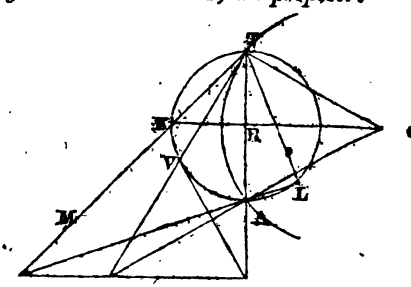
line of the fourth order. And various other answers are received from Messrs. Webster, Johnson, Ambulator, Maffet, and Shenstone.

It may not be improper to remark here, that the general problem, of which question 5, Ladies' Diary 1817, is a particular case, admits of an elegant solution by means of the above *locus:* for it is evident that if the hyperbola which is the locus of the vertex C, be described, it will cut or touch the semicircle in some points C,C or point C, which determines the position and length of the required chord. EDIT.

(156) *By* Mr. J. H. SWALE, *Liverpool.*

Given the tangent and chord of a circular arc to determine it.

Answered by Mr. J. H. SWALE, *the proposer.*



COMP. Upon the given chord AT, as a diameter, describe a circle, draw the radius RK perpendicular to AT, join T, K, and produce TK making TP × PK = TM² the square of the given tangent: draw PG perpendicular to TA, make GB = GV, a tangent to the circle on AT; join B, A, meeting KR in C, the centre of the required arc. Join CT, BT: since RA = RT, and RC perpendicular to AT, we shall have CA = CT, or the circle through A, to the centre C, passes through T. To centre C and radius CA or CT, describe the circle: then AT being equal to the given chord, it remains to shew that BT touches the arc AT at T, and that it is equal the given tangent TM. Join PA, meeting the circle on AT in L: By the COMP. GB = GV, or GB²

$= GV^2 = GT.GA$, it follows that, $\angle GBA = \angle GTB = \angle ACR = \angle TCR$, and of course $\angle GTB + \angle GTC = \angle RCT + \angle GTC =$ a right $\angle$; or $\angle CTB =$ a right $\angle$; that is, BT is a tangent at T. Again, since $\angle TGB$ is a right $\angle$, and $TG = GP$, we shall have $BT^2 = BG^2 + GT^2 = BG^2 + GP^2 = GT.GA + GP^2 = GA^2 + GP^2 + GA.AT = PA^2 + PA.AL = PL.PA = PT.PK$ per the circle $= TM^2$ by comp. that is, BT = TM the given tangent.

The points T, V, B, are in a right line: we leave the demonstration of this to the young Geometer. This problem is the same as the hypothenuse and the line bisecting one of the acute $\angle$s, of a right angled $\triangle$, being given, to determine it: to which problem, as well as to the above, a variety of elegant comp. may be given; the limited pages of the Correspondent preclude their insertion.

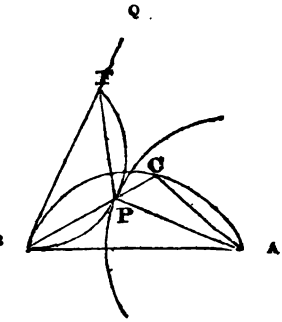
Thus nearly is the answer given by Messrs. Postlethwaite, Whitehead, Garside, Johnson, Webster, Sollitt, Lenthwaite, Verus, and Ambulator.

(157) *By* Mr. Wm. Shillito, *Manchester.*

Given the base AB, and the vertical angle C, to construct the plane triangle ACB so that drawing AP meeting BC in P, BP shall have to AC a given ratio, and AP equal a given line, *a maximum* or *minimum.*

Answered by the Proposer.

Const. On the given base AB describe a circular segment to contain the given vertical angle, and draw BQ a tangent to the circle at B: in BQ take BF so that AB may be to BF in the given ratio of AC to BP: on BF describe a segment of a circle similar to ACB: and from the centre A with the given distance AP describe a circle cutting or touching FPB in P: through P draw BC meeting BCA in C: join AC: then is ACB the required triangle.



DEM. The base and vert. $\angle$ are $=$ to the given ones by the Construction. Also $\angle$ FPB $=$ BCA, and (Eu. 32. 3.) $\angle$ FBP $=$ BAC: therefore the $\triangle$s, FBP, BAC are similar; hence AB : AC :: FB : BP, Altern. AB : BF :: AC : BP, the given ratio; also AP is equal to the given line by Construction. Q. E.D.

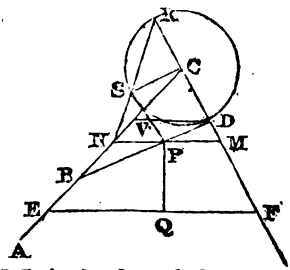
When the circle described from the centre A touches the segment FPB it is evident that AP is then a *minimum*.

Answers were also received from Messrs. Treeby, Johnson, and Swale.

(158) *By* Mr. J. H. SWALE.

The lines AB, CD, EF are given in position; between two of these lines, as AB, CD, it is required to interpose a given line BD so that the perpendicular PQ from a given point P in BD, upon the remaining line EF, shall equal a given line a *maximum* or *minimum*.

Answered by VERUS, *from London.*



CONST. Draw NM parallel to EF at the given perpendicular distance PQ meeting CB in N and CD in M : take m = BP, and n = DP : find MR a fourth proportional to m, $m + n$, and CM : join NR : and from the centre C, with the radius CS $= n$, describe a circle cutting or touching N R in S : from S draw SP parallel to CD cutting NM in P ; through P draw BD parallel to SC, and the thing will be done.

DEM. Draw DV || to EF meeting BC in V. By similar △s,

$$MN : RM :: PN : PS = CD$$
$$BD : BP :: DV : PN$$
$$CM : NM :: CD : DV$$

Whence comp. $BD \times CM : BP \times RM :: 1 : 1$
Therefore $BP : BD :: CM : RM.$
But $m : m + n :: CM : RM$, by const.
Ergo ex equali $BD : BP :: m + n : m$
Dividendo $DP : BP :: n : m$

But since SCDP is a parallelogram $SC = PD = n$ (const.): consequently $BP = m$ and PQ is the given perpendicular by construction. Q.E.D.

When PQ is a minimum NM will be a maximum, which is evidently the case when NR is a tangent to the circle at S.

If we suppose the given line BD to revolve between the straight lines CB, CD given in position, the given point P in that line will describe an ellipse, as is well known. And if a tangent be drawn to this ellipse parallel to EF the nearest distance between the two parallels will be equal to PQ the minimum. Hence it follows that when NR is a tangent to the circle whose radius is SC, NM will be a tangent to the ellipse which is the locus of the point P. And this is a very curious and elegant property worthy the attention of the young geometer.

Ingenious solutions to this question were also received from Messrs. Treeby, Johnson, and Swale.

(159) By AMICUS.

If P be the point in a polygon from which perpendiculars are drawn to the sides of the polygon so that the sum of their squares is the least possible; twice the area of the polygon is a mean proportional between the sum of the squares of the sides of the polygon and the sum of the squares of the abovementioned perpendiculars. Required the demonstration or investigation.

Answered by AMICUS, *the proposer.*

Let $a, b, c, d, e,$ &c. to n terms denote the sides of the polygon, and $u, y, z, v, w, \ldots\ldots n$ the respective perpendiculars falling on them from the point P. Put A = twice the area of the polygon,

$$S = a^2 + b^2 + c^2 + d^2 + e^2 + \ldots\ldots\ldots n$$
$$P = u^2 + y^2 + z^2 + v^2 + w^2 + \ldots\ldots\ldots n.$$

Imagine lines drawn from the point P to the angles of the polygon dividing it into n triangles: then it is evident that twice the area of the polygon,

$$\text{or } A = au + by + cz + dv + ew + \ldots\ldots\ldots n \quad (1)$$

But $P = u^2 + y^2 + z^2 + v^2 + w^2 + \ldots n$ (2) a min.

Therefore taking the fluxions of these two equations, with u constant, we have

$$\dot{b}y + c\dot{z} + d\dot{v} + e\dot{w} + \ldots\ldots\ldots n = 0. \quad (3)$$

$$\dot{y}y + \dot{z}z + \dot{v}v + \dot{w}w + \ldots\ldots\ldots n = 0. \quad (4)$$

Multiply No 3 by y and No 4 by b, and we shall have

$$\dot{b}yy + y\ (c\dot{z} + d\dot{v} + e\dot{w} + \ldots\ldots n) = 0 \quad (5)$$

$$\dot{b}yy + b\ (\dot{z}z + \dot{v}v + \dot{w}w + \ldots\ldots n) = 0. \quad (6)$$

Equating the two equations 5 and 6 we readily deduce

$$y = b\ (\dot{z}z + \dot{v}v + \dot{w}w + \ldots\ldots\ldots n) \div (c\dot{z} + d\dot{v} + e\dot{w} + \ldots\ldots\ldots n) \quad (7)$$

Again, taking the fluxions of the equations 1 and 2, with y only invariable we get,

$$a\dot{u} + c\dot{z} + d\dot{v} + e\dot{w} + \ldots\ldots\ldots n = 0. \quad (8)$$

$$u\dot{u} + z\dot{z} + v\dot{v} + w\dot{w} + \ldots\ldots\ldots n = 0. \quad (9)$$

Multiply No 8 by u and No 9 by a, and there will result

$$a\dot{u}u + u\ (c\dot{z} + d\dot{v} + e\dot{w} + \ldots\ldots\ldots n) = 0 \quad (10)$$

$$a\dot{u}u + a\ (\dot{z}z + \dot{v}v + \dot{w}w + \ldots\ldots\ldots n) = 0 \quad (11)$$

Equating Nos 10 and 11, we find $u = a\ (\dot{z}z + \dot{v}v + \dot{w}w + \ldots\ldots\ldots n) \div (c\dot{z} + d\dot{v} + e\dot{w} + \ldots\ldots\ldots n)$ (12)

Comparing this value of u with that of y No 7, we readily perceive that $u : y :: a : b$; so that the perpen-

diculars u, y, z, v &c. are directly proportional to the sides of the polygon on which they fall; or $y : z :: b : c$, $z : v :: c : d$, $v : w :: d : e$, &c. therefore $y = \frac{bu}{a}$, $z = \frac{cy}{b} = \frac{cu}{a}$, $v = \frac{dz}{c} = \frac{du}{a}$, $w = \frac{ev}{d} = \frac{eu}{a}$ &c. Hence $au + by + cz + dv + \ldots\ldots n =$

$$= au + \frac{b^2u}{a} + \frac{c^2u}{a} + \frac{d^2u}{a} + \ldots n = A.$$

Therefore $u = Aa \div (a^2 + b^2 + c^2 + d^2 + \ldots\ldots n)$

$$= \frac{Aa}{S}, \; y = \frac{Ab}{S}, \; z = \frac{Ac}{S}, \; v = \frac{Ad}{S}, \; w = \frac{Ae}{S} \text{ &c.}$$

Whence $u^2 + y^2 + z^2 + v^2 + w^2 + \ldots\ldots\ldots n =$

$$= (a^2 + b^2 + c^2 + d^2 + e^2 + \ldots\ldots \ldots n). \frac{A^2}{S^2} = \frac{A^2}{S}$$

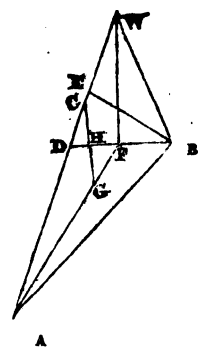
$= P$. Consequently $A^2 = S \times P$, or $S : A :: A : P$. that is twice the area of the polygon &c. Q.E.I.

Mr. Johnson also favoured us with a solution.

(160) *By the same.*

If a conical vessel, of no sensible weight, whose diameter is 4 and slant height 10 inches, be freely suspended by a point in the side 9 inches from the vertex: how much water will it hold when at rest?

Answered by Mr. S. JOHNSON.



Let ACEB, be a section of the glass passing through its axis; DB the surface of the water. Bisect DB in F, and join AF, also fromC, the point of suspension, let fall on DB the perpendicular CH, meeting AF at G, the centre of gravity of the contained fluid, and from F draw FW parallel to CH, meeting the side of the cone produced in W. Then by the property of the centre of gravity AG : GF :: 3 : 1 :: (by parallel lines) AC = 9 : CW = 3;

nce WE = 2. Join BW. Now because DF = ', and FW is perpendicular to DB, we have DW = N. But, by page 128 Simpson's Exercises, BW^2 or $WE^2 = \frac{BE^2 \times AW + AE \times EW \times AW - AB^2}{AE}$

$\frac{EW}{\quad}$; which (because AE = AB) $= \frac{BE^2 \times AW}{AE}$

WE^2; hence $DW = BW = \sqrt{\frac{BE^2 \times AW}{AE} + WE^2}$

$= \sqrt{\left(\frac{16 \times 12}{10} + 4\right)} = \sqrt{23.2} = 4.816637$; there-

re CD = DW − CW = 4.816637 − 3 = 1.816637, so DE = 2.816637; from which the content of the ngula DEB = 16.874768. Also the content of the lass = 41.041514, and the difference is 24.666746 = ie content of ADB as required.

Thus nearly is the answer given by Messrs. Maffett, 'reeby, Webster, Lewthwaite, and Verus.

(161) *By* Mr. ROBERT SMITH, *Manchester.*

A gentleman has named a horse for two gold cups: ow the odds are 2 to 1 that he runs for the first, and rovided he runs, 3 to 2 in his favour: also 3 to 2 iat he runs for the second, and if he does run, 5 to 4 iat he wins. It is required to determine the odds in ivour of the horse absolutely winning one cup precisely or both.

Answered by Mr. R. SMITH, *the Proposer.*

It is clear, that to win one precisely, one of these ur compound events must take place, viz.

1. He must win the first and lose the second; or
2. lose the first, and win the second; or
3. win the first, and draw for the second; or
4. draw for the first, and win the second.

Hence, per question, these four probabilities will re-ectively stand thus: 1. $\frac{2}{3} \times \frac{2}{5} \times \frac{3}{5} \times \frac{1}{2} = \frac{6}{75}$

2. $\frac{2}{3} \times \frac{3}{5} \times \frac{2}{5} \times \frac{1}{2} = \frac{4}{75}$

3. $\frac{2}{3} \times \frac{5}{6} \times \frac{2}{5} = \frac{4}{45}$
4. $\frac{1}{2} \times \frac{5}{6} \times \frac{5}{6} = \frac{25}{72}$

Then $\frac{8}{15} + \frac{4}{45} + \frac{4}{45} + \frac{1}{5} = \frac{7}{15}$ the probability of winning one precisely; but the probability of winning both is $\frac{2}{3} \times \frac{3}{5} \times \frac{3}{5} \times \frac{5}{6} = \frac{3}{15}$ hence $\frac{7}{15} + \frac{3}{15} = \frac{2}{3}$ the probability of absolutely winning one precisely, or both.

Therefore $1 - \frac{2}{3} = \frac{1}{3}$ the probability of the contrary, Consequently the odds are 3 to 2 in favour of him absolutely winning one cup precisely, or both.

Again by Mr. L. SYRAY.

By Simpson's chances, Law First, $\frac{2}{3}$ is the probability of running for the first cup, and $\frac{3}{5}$ for the second, and $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$ the probability that he runs for both. Also $\frac{3}{5} =$ the probab. of his winning the first if he run and $\frac{5}{6} =$ —— —— —— second —— hence $\frac{2}{3} \times \frac{5}{6} = \frac{5}{9} =$ prob. of his actually winning the first and $\frac{2}{5} \times \frac{5}{6} = \frac{1}{3} =$ —— —— —— second therefore $\frac{5}{9} \times \frac{1}{3} = \frac{5}{15} =$ —— —— both Hence the odds are as 2 to 13; again the probability of his losing the first is $\frac{3}{5}$, hence $\frac{3}{5} \times \frac{1}{3} = \frac{1}{5}$ his expectation of the second, which being added to $\frac{2}{5}$ the expectation in the first gives $\frac{3}{5}$ for the probability of his winning one precisely. Hence the odds are as 3 to 2.

Or thus: the probability of losing in the first $\frac{2}{5}$, by the same in the second (viz.) $\frac{2}{5} = \frac{4}{25} =$ the probability of losing in both cases; hence $1 - \frac{4}{25} = \frac{21}{25}$ as before.

True and ingenious answers were also given by Messrs. Johnson, Lewthwaite, and Verus.

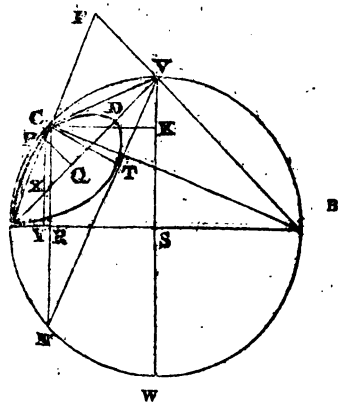
(162) *By* Mr. SAMUEL JOHNSON.

Let AB be the diameter of a given semicircle ACB, and in any chord AC let AP be always taken equal to the corresponding right sine CR. It is required to find the length of the curve described by the Point P.

Also to determine, geometrically, the greatest ordinate to the axis.

See the solutions of question 125 where the quadrature of this curve is determined.

Answered by Mr S. JOHNSON, *the Proposer.*



First, the diameter of the curve will be (supposing the centre S V = radius perpendicular to AB, and AV joined) in AV, and equal to AS. For when CR = SV, then will AP = SV. Join BV, and let it meet AC produced to F; also draw PQ perpendicular to AV. Put AV = V = a, AB = b, Q = x, PQ = y, then $a : y :: a : \frac{ay}{x}$ = VF; hence BF = $\frac{a}{x}(x+y)$; and AF = $\sqrt{\quad}$ $x^2 + y^2$; CF = $\frac{ay\,(x+y)}{x\sqrt{x^2+y^2}}$ whence AC = $\frac{a(x-y)}{\sqrt{x^2+y^2}}$

also AR = $\frac{a^2(x-y)^2}{b\,(x^2+y^2)}$; BR = $b - \frac{a^2\,(x-y)^2}{b\,(x^2+y^2)}$ But AR

BR = CR^2 = AP^2 = $\frac{a^2(x-y)^2}{(x^2+y^2)} - \frac{a^4(x-y)^4}{b^2(x^2+y^2)^2}$ =

$+y^2$; hence $b^2(x^2+y^2)^3 - a^2b^2(x-y)^2 \times (x^2+y^2)$

$- a^4(x-y)^4 = 0$; but $b^2 = 2a^2$; hence by substitution $2a^2(x^2+y^2)^3 - 2a^4(x-y)^2 \times (x^2+y^2) + a^4$

$)-y)^4 = 0$. This reduces to $2(x^2+y^2)^3 = a^2(x^2 -$

$)^2$; let $x^2+y^2 = z^2$; and $x^2 - y^2 = z^2 - 2y^2$: hence $2z^6$

$= a^2(z^2 - 2y^2)^2$, from which $2ay^2 = az^2 - z^3\sqrt{2}$,

and $y^2 = \frac{az^2 - z^3\sqrt{2}}{2a}$, $x^2 = \frac{az^2 + z^3\sqrt{2}}{2a}$; let AS = r,

then $a = r\sqrt{2}$; hence $y = \frac{\sqrt{rz^2 - z^3}}{\sqrt{2r}}$, $x = \frac{\sqrt{rz^2 + z^3}}{\sqrt{2r}}$;

therefore $\dot{y} = \frac{2\dot{r}z - 3z\dot{z}}{2\sqrt{2r}\sqrt{(r-z)}}$, $\dot{x} = \frac{2r\dot{z} + 3z\dot{z}}{2\sqrt{2r}\sqrt{(r+z)}}$;

hence $\dot{x}^2 + \dot{y}^2 = \frac{(8r^3 - 6rz^2)}{8r(r^2 - z^2)} \cdot \dot{z}^2 = \left(\frac{4r^2 - 3z^2}{4(r^2 - z^2)}\right) \times \dot{z}^2$;

conseq. $\sqrt{\dot{x}^2 + \dot{y}^2} = \frac{\dot{z}}{2} \frac{\sqrt{(4r^2 - 3z^2)}}{\sqrt{(r^2 - z^2)}} = \dot{z} \frac{\sqrt{(r^2 - \frac{3}{4}z^2)}}{\sqrt{(r^2 - z^2)}}$;

put $rw = z$, then $\dot{z} \frac{\sqrt{(r^2 - \frac{3}{4}z^2)}}{\sqrt{(r^2 - z^2)}} = r\dot{w} \frac{\sqrt{(1 - \frac{3}{4}w^2)}}{\sqrt{(1 - w^2)}}$

Now the fluent of $\dot{w} \frac{\sqrt{(1 - \frac{3}{4}w^2)}}{\sqrt{(1 - w^2)}}$ is, by page 237 Hodson's Fluxions, an elliptic arc contained between the semi-conjugate $\frac{1}{2}$, and a parallel ordinate at the distance w from the centre. The semi-transverse being 1, which when $w = 1$ becomes the quadrant of an ellipsis whose conjugate and transverse diameters are 1 and 2; hence the whole length of the oval $= r \times$ semi-ellipsis, the transverse and conjugate diameters being 2 and 1.

To determine the greatest ordinate. Let the circle be completed and produce CR, VS to meet the circumference again at N and W; join CV, VN, and draw CK, CT, perpendicular to VW, VN respectively. Then the angle PAQ = CNT, and the angles AQP, CTN, right angles, the triangles AQP, CTN are similar; but NC = 2AP per quest. therefore CT = 2PQ = a max. or the rectangle CT × VW = NC × CV = a max. or its half CV × CR = CV × SK, or its square $CV^2 \times SK^2 = 2VS \times VK \times SK^2$, but 2VS being given we have VK × SK^2 = a max. hence by theor. 17 *Simpson's Max. et. Min.* SK = 2VK.

Otherwise by AMICUS.

Draw PY perpendicular to AB; and put AB = 2r, Ay = x, and Py = y, then AP = CR = $\sqrt{(x^2 + y^2)}$; and by similar triangles $y : x :: \sqrt{(x^2 + y^2)} : \frac{x}{y}\sqrt{(x^2 + y^2)} = AR$: and $x : y :: \sqrt{(x^2 + y^2)} : \frac{y}{x}\sqrt{(x^2 + y^2)} = BR$;